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Virtual Sensors Design for Nonlinear Dynamic Systems

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ABSTRACT

The objective of the paper is virtual sensors design, estimating prescribed components of the systems state vector to solve the tasks of fault diagnosis in nonlinear systems. To solve the problem, the method called logic-dynamic approach is used, which allows to solve the problem for systems with non-smooth nonlinearities subjected to external disturbance by methods of linear algebra. According to this method, the problem is solved in three steps: at the first step, the nonlinear term is removed from the system, and the linear model invariant with respect to the disturbance is designed; at the second step, a possibility to take into account the nonlinear term and to estimate the given variable is checked; finally, the transformed nonlinear term is added to the linear model. The relations allowing to design virtual sensor of minimal dimension estimating prescribed component of the state vector of the system are obtained. The main contribution of the present paper is that a procedure to design virtual sensors of minimal dimension for nonlinear systems estimating prescribed components of the state vector is developed. This allows to reduce complexity of the virtual sensors in comparison with known papers where such sensors of full dimension are constructed. Besides, the limitations imposed on the initial system are relaxed that allow to extend a class of systems for which the virtual sensors can be constructed.

1. Introduction

Modern complex technical systems have different sensors; they are used, in particular, to control and fault diagnosis. Clearly, the more sensors are used, the simpler solution is obtained. Additional physical sensors result in extra expenses; besides, they are of no high reliability. Here, virtual sensors are of interest. There are many papers considering different problems in the design and application of virtual sensors. Most of these papers are intended to solve different practical applications of virtual sensors: for health monitoring of automotive engines [1]; for active reduction of noise in active control systems [3]; for electronic nose devises [5]; for hiding the fault from the controller point of view [7]; in walking legged robots [8]; for failure diagnosis in aircraft [9]; in the process of fault detection in industrial motor [10]; for fault detection, isolation and data recovery in a bicomponent mixing machine [11]; in humidity sensor systems [14]; in the sensor-cloud platform [17]; for a tunnel furnace [27]. In [16], [18]-[21], [28], virtual sensors are used for fault tolerant control for different types of dynamic systems (descriptor, parameter varying, subject to actuator saturation, etc.). A new architectural paradigm for remotely deployed sensors whereby a sensor's software is separated from the hardware is presented in [24]. In [2], [13], different theoretical aspects of using virtual sensors in linear systems are considered. A detailed procedure to design virtual sensors for linear systems is suggested in [4]. The main gap of all considered papers is that they design virtual sensors of full dimension for systems described by linear models; besides, such sensors are subjected to external disturbance.

The main contribution of the research is that a procedure to design virtual sensors of minimal dimension for nonlinear systems with non-smooth nonlinearities estimating prescribed components of the state vector and insensitive to the external disturbance is developed. This allows to reduce complexity of the virtual sensors in comparison with cited above papers where such sensors of full dimension are constructed. Besides, the limitations imposed on the initial system are relaxed that allow to extend a class of systems for which the virtual sensors can be constructed.

The set of the prescribed components depends on the problem of control or fault diagnosis under consideration. Here we consider the problem of sensor fault identification based on sliding mode observers. The use of virtual sensors allows to improve a solution of this problem.

2. The Model Design

The problem of virtual sensor design will be solved for systems described by a continuous-time model as in (1).

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) + P\phi(Ax(t), u(t)) + L\rho(t) \\ y(t) &= Hx(t) \end{aligned} \tag{1}$$

and discrete-time model shown in (2).

$$\begin{aligned} x(t+1) &= Fx(t) + Gu(t) + P\phi(Ax(t), u(t)) + L\rho(t) \\ y(t) &= Hx(t) \end{aligned} \tag{2}$$

where $x(t) \in R^n$, $u(t) \in R^m$ и $y(t) \in R^l$ are state, control, and output vectors, F, G, H, C, P, and L are known matrices; it is assumed that the disturbance $\rho(t) \in R^p$ is an unknown bounded function of time; for simplicity, we assume that the only type of nonlinearity described by the term $\phi(Ax(t), u(t))$ is in the system, A is the matrix, the function ϕ maybe non-smooth.

The problem is as follows: given $v(t) = H_v x(t)$ for known matrix H_v, construct a virtual sensor of minimal dimension estimating the variable $v(t)$. Assume for simplicity that H_v is a row matrix.

To design the virtual sensor, so-called logic-dynamic approach is used, which allows to solve the problem for nonlinear systems by linear methods. According to this approach, the problem is solved in three steps: in the first step, the nonlinear term is removed from (1) and (2), and the linear model of minimal dimension invariant with respect to the disturbance and estimating the variable $v(t)$ under some additional conditions is designed; at the second step, a possibility to take into account the nonlinear term and to estimate the given variable is checked; finally, the transformed nonlinear term is added to the linear model.

The linear model of minimal dimension invariant with respect to the disturbance $\rho(t)$ designed at the first step is given by (3).

$$\begin{aligned} \dot{z}(t) &= F_* z(t) + K_* y(t) + G_* u(t) \\ v(t) &= Cz(t) + Qy(t) \end{aligned} \tag{3}$$

for the continuous-time case, as in (4).

$$\begin{aligned} z(t+1) &= F_* z(t) + K_* y(t) + G_* u(t) \\ v(t) &= Cz(t) + Qy(t) \end{aligned} \tag{4}$$

for the discrete-time one, where$z(t) \in R^k$, $k < n$, is the state vector, F_*, G_*, C, Q, and K_* are matrices to be determined.

It is assumed that the relation $z(t) = \Psi x(t)$ is true where Ψ is some constant matrix to be determined. It is known [29] that it meets the following Equations (5).

$$\Psi F = F_* \Psi + K_* H, \quad G_* = \Psi G \tag{5}$$

The first additional condition appears due to Equation (6).

$$v(t) = H_v x(t) = Cz(t) + Qy(t) \tag{6}$$

Since $z(t) = \Psi x(t)$ and $y(t) = Hx(t)$, it follows (7).

$$H_v = C\Psi + QH = \begin{pmatrix} C & Q \end{pmatrix} \begin{pmatrix} \Psi \\ H \end{pmatrix} \tag{7}$$

This equation has a solution if and only if the matrix H_v can be expressed via the matrices Ψ and H that is equivalent to (8).

$$\operatorname{rank} \begin{pmatrix} \Psi \\ \mathrm{H} \end{pmatrix} = \operatorname{rank} \begin{pmatrix} \Psi \\ \mathrm{H} \\ \mathrm{H_v} \end{pmatrix} \tag{8}$$

The second additional condition appears due to the nonlinear term. This term in the nonlinear model is of the form (9).

$$P_* \phi(A_* \begin{pmatrix} z(t) \\ y(t) \end{pmatrix}, u(t)) \tag{9}$$

where $P_* = \Psi P$, the matrix A_* satisfies the condition (10) [30], [31].

$$A = A_* \begin{pmatrix} \Psi \\ H \end{pmatrix} \tag{10}$$

This equation has a solution if and only if, as in (11).

$$\operatorname{rank} \begin{pmatrix} \Psi \\ H \end{pmatrix} = \operatorname{rank} \begin{pmatrix} \Psi \\ H \\ A \end{pmatrix} \tag{11}$$

To simplify the solution of Equations (5), the matrix F_* will be sought in a canonical form different from the continuous-time and discrete-time models. Consider at first the discrete-time case.

2.1. Discrete-time Model

To construct a model (4), the matrix F_* is sought in the identification canonical form (12).

$$F_* = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \tag{12}$$

It was shown that such a form always exists [29]. This matrix has zero eigenvalues; therefore, it is stable. Therefore there is no need to use special feedback for achieving stability in the linear discrete-time model (4). Besides, it enables to obtain simple equations for matrices describing the models (4) as follows [29]. Using (12), one obtains from (5) the following Equations (13).

$$\Psi_i F = \Psi_{i+1} + K_{*i} H, \;\; i = 1, \ldots, k-1, \;\; \Psi_k F = K_k H \tag{13}$$

the i-th rows of the matrices Ψ and K_*, $i = 1,\dots,k$, are denoted by Ψ_i and K_{*i}, respectively.

The model (4) is insensitive to $\rho(t)$ when $\Psi L = 0$; as a result, Equations (13) are transformed into (14).

$$(\Psi_1 \quad -K_{*1} \quad \dots \quad -K_{*k})(V^{(k)} \quad L^{(k)}) = 0 \tag{14}$$

where as in (15).

$$V^{(k)} = \begin{pmatrix} F^k \\ HF^{k-1} \\ \vdots \\ H \end{pmatrix}, \ L^{(k)} = \begin{pmatrix} L & FL & \dots & F^{k-1}L \\ 0 & HL & \dots & HF^{k-2}L \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \tag{15}$$

Note that the equality $(\Psi_1 \quad -K_{*1} \quad \dots \quad -K_{*k})V^{(k)} = 0$ allows to construct the model (4), the equality $(\Psi_1 \quad -K_{*1} \quad \dots \quad -K_{*k})L^{(k)} = 0$ ensures insensitivity to the disturbance.

This equation should be solved for minimal k starting from $k = 1$. Then the rows of matrix Ψ are found from (11); as a result, the first step has been finished. In the second step, the conditions (8) and (11) are checked; if they are fulfilled, the matrices C, Q, and A_* are found from (7) and (10), respectively; $P_* = \Psi P$. The nonlinear term (9) is added to the linear model (4); as a result, the nonlinear model has been designed in the form (16).

$$z(t+1) = F_* z(t) + K_* y(t) + G_* u(t) + P_* \phi(A_* \begin{pmatrix} z(t) \\ y(t) \end{pmatrix}, u(t)) \tag{16}$$

If conditions (8) and (11) are not fulfilled, one finds another solution of (14) with former or incremented dimension k. If (14) is not satisfied for all $k < n$, the virtual sensor invariant with respect to the disturbance cannot be designed; one has to use the robust methods [30].

2.2. Continuous-time Model

To construct a model (3), the matrix F_* is sought in the purely diagonal Jordan canonical form with different eigenvalues $\lambda_1,\dots,\lambda_k$, so that the form as in (17) is obtained.

$$F_* = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_k \end{pmatrix} \tag{17}$$

Clearly, the matrix (17) with $\lambda_i < 0$ ensures stability of the continuous-time model. Here k independent equations present the equation $\Psi F = F_* \Psi + K_* H$ as shown in (18).

$$\Psi_i F = \lambda_i \Psi_i + K_{*i} H, \quad i = 1,\dots,k \tag{18}$$

Take into account insensitivity to the disturbance when $\Psi L = 0$. Define the matrix L_0 of maximal rank satisfying the condition $L_0 L = 0$. As a result, $\Psi = N L_0$ for some matrix N, and (18) can be rewritten in the form (19).

$$(N_i \quad -K_{*i}) \begin{pmatrix} L_0(F - \lambda_i I_n) \\ H \end{pmatrix} = 0, \ \ i = 1,\dots,k \tag{19}$$

where I_n is the identical matrix.

To design the virtual sensor for continuous-time systems, one chooses some $\lambda_i < 0$ and finds $\Psi_i = N_i L_0$ form (19) such that the matrix Ψ satisfies the conditions (8) and (11); then matrices C, Q, and A_* are found from (7) and (10) respectively. If the matrix Ψ does not exist, the virtual sensor invariant with respect to the disturbance cannot be designed, and one has to use robust methods [30]. The nonlinear model has been designed in the form (20).

$$\dot{z}(t) = F_* z(t) + K_* y(t) + G_* u(t) + P_* \phi(A_* \begin{pmatrix} z(t) \\ y(t) \end{pmatrix}, u(t)) \tag{20}$$

The next algorithm demonstrates the main steps to design the virtual sensor; to be specific, consider the discrete-time system.

2.3. Algorithm

1. Solve the equation (14) for minimal k stating from $k = 1$, find the matrices Ψ_1 and K_*, and the other rows of the matrix Ψ based on (13).
2. Check the conditions (8) and (11); if they both are satisfied, find the matrices C, Q, and A_* from (7) and (10), respectively. Otherwise, find another solution of (14) with former or incremented k. If (8) or (11) is not satisfied for all $k < n$, the virtual sensor invariant with respect to the disturbance cannot be designed; one has to use the robust methods [30].
3. Set $G_* = \Psi G$, $P_* = \Psi P$, and construct the model (16).

3. Stability

Because of the matrix F_* choice, the linear models are stable. In some cases, the nonlinear term does not change stability, consider this in detail.

Introduce the estimation error $e(t) = \Psi x(t) - z(t)$ and write down the equation for $e(t)$ taking into account relations (5) (consider for simplicity the continuous-time case only), so that obtained (21).

$$\begin{aligned} \dot{e}(t) &= \Psi(Fx(t) + Gu(t) + P\phi(Ax(t), u(t))) - \\ &- (F_* z(t) + K_* y(t) + G_* u(t) + P_* \phi(A_* \begin{pmatrix} z(t) \\ y(t) \end{pmatrix}, u(t))) = F_* e(t) - \Delta\phi(t) \end{aligned} \tag{21}$$

where shown in (22).

$$\Delta\phi(t) = \Psi P \phi(Ax(t), u(t)) - P_* \phi(A_* \begin{pmatrix} z(t) \\ y(t) \end{pmatrix}, u(t)) \tag{22}$$

It is assumed that the function $P\phi(Ax, u)$ satisfies the Lipschitz condition about x uniformly for u obtained as in (23).

$$||P\phi(Ax, u) - P\phi(Ax', u)|| \leq M||x' - x|| \tag{23}$$

where $M > 0$ is some constant. Then $\Delta\phi(t)$ satisfies similar condition as well: $||\Delta\phi(t)|| \leq M_*||e(t)||$ for some $M_* > 0$. Since the matrix F_* is stable, the symmetric positive definite matrices P and W exist such that obtained as in (24).

$$F_*^T P + P F_* = -W \tag{24}$$

Consider Lyapunov candidate function $V(t) = e^T(t) P e(t)$ and take its derivative using (23) and (24) so obtain a new form (25).

$$\begin{aligned} \dot{V}(t) &= (F_* e(t) - \Delta\phi(t))^T P e(t) + e^T(t) P (F_* e(t) - \Delta\phi(t)) \\ &= e^T(t)(F_*^T P + P F_*) e(t) + 2e^T(t) P \Delta\phi(t) \\ &= -e^T(t) W e(t) + 2e^T(t) P \Delta\phi(t) \\ &\leq -||e(t)||^2 (\lambda * max_{min} \end{aligned} \tag{25}$$

Clearly, if as in (26).

$$M_* < \frac{\lambda_{min}}{2\lambda_{max}} \tag{26}$$

then $\dot{V}(t) < 0$, and the observer is stable. Note that this approach is considered in [15]. If (26) is not satisfied, stability can be achieved by methods suggested in [15].

4. Virtual Sensor Location

The choice of the matrix H_v depends on the problem under consideration. Consider as an example the problem of sensor fault identification based on sliding mode observers. Note that to solve this problem, the methods suggested in [6], [12], [23], [26] provide only approximate solutions to this problem since the final expressions contain the derivative $\dot{d}(t)$ where the unknown function $d(t)$ describes the fault.

The method suggested in [6] assumes that a new state vector being a filtered version of $y(t)$ is introduced and special system of the dimension $n + l$ is constructed. Then, based on this system, the sliding mode observer is designed under some restrictions imposed on the original systems.

A new method of solving this problem was developed in [22], [31], [32]. Similar to the developed above method of virtual observer design, it is based on the model of minimal dimension invariant with respect to the disturbance. In contrast to the methods suggested in [6], [12], [22], it allows to design sliding mode observer of the dimension $k < n$ which does not contain the derivative $\dot{d}(t)$ and provides the exact solution.

Note that to design a sliding mode observer by method [22], the additional condition $R_* D = 0$ should be taken into account; otherwise, siding motion cannot be obtained. Here D is the matrix describing the fault in the sensor according to (27).

$$y(t) = Hx(t) + Dd(t) \tag{27}$$

To take into account the condition $R_* D = 0$, introduce the matrix D_* of maximal rank such that $D_* D = 0$, then $R_* = SD_*$ for some matrix S. By analogy with (12), the model is designed based on Equation (28).

$$(S \quad -K_{*1} \quad \dots \quad -K_{*k})(V^{s(k)} \quad L^{s(k)}) = 0 \tag{28}$$

where shown in (29).

$$V^{s(k)} = \begin{pmatrix} D_* F^k \\ HF^{k-1} \\ \vdots \\ H \end{pmatrix}, \quad L^{s(k)} = \begin{pmatrix} D_* HL & D_* HFL & \dots & D_* HF^{k-1} L \\ 0 & HL & \dots & HF^{k-2} L \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \tag{29}$$

As above, this equation is solved for minimal k, and the model and then sliding mode observer are designed.

Rules to choose the virtual sensor H_v are as follows. Let D_0 be the matrix corresponding to the faulty sensor in the extended (with this sensor) output vector $y_0(t)$ and D_{0*} be the matrix of maximal rank such that $D_{0*} D_0 = 0$. Taking into account (29), we conclude that the matrix H_v should be chosen such that (30).

$$\text{rank}(D_{0*} H) < \text{rank}\left(D_{0*} \begin{pmatrix} \Psi \\ H_v \end{pmatrix}\right) \tag{30}$$

since such a choice yields more possibilities to find the matrix R_* satisfying the condition $R_* D_0 = 0$. When two variants H_{v1} and H_{v2} are possible, the first one is preferable if, as in (31).

$$\mathrm{rank}\left(D_{0*1}\begin{pmatrix}\Psi \\ H_{v1}\end{pmatrix}\right) > \mathrm{rank}\left(D_{0*2}\begin{pmatrix}\Psi \\ H_{v2}\end{pmatrix}\right) \tag{31}$$

where the matrices D_{0*1} and H_{v1} correspond to the first variant, D_{0*2} and H_{v2} to the second, respectively.

Another variant of the matrix H_v choice can be motivated as follows. The simplest solutions of fault diagnosis problems are when all components of the state vector $x(t)$ are available. This yields the rule in the form (32).

$$\mathrm{rank}\begin{pmatrix}H \\ H_v\end{pmatrix} = n \tag{32}$$

5. Example

Consider the general electric servo actuator described by equations (33).

$$\begin{aligned}\dot{x}_1(t) &= k_1x_2(t) + \rho(t)\\ \dot{x}_2(t) &= k_2x_2(t) + k_3x_3(t) + k_7sign(x_2(t))\\ \dot{x}_3(t) &= k_4x_2(t) + k_5x_3(t) + k_6u(t)\end{aligned} \tag{33}$$

where $x_1(t)$ is the output rotation angle at the reducer output shaft; $x_2(t)$ is the output rotation velocity at the motor output shaft; $x_3(t)$ is the current through the servo actuator windings; $u(t)$ is a voltage. The coefficients $k_1 \div k_6$ depend on the parameters of the servo actuator. The servo actuator is described by matrices and nonlinearities as in (34).

$$F = \begin{pmatrix}0 & k_1 & 0\\ 0 & k_2 & k_3\\ 0 & k_4 & k_5\end{pmatrix},\ G = \begin{pmatrix}0\\0\\k_6\end{pmatrix},\ H = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix},\ P = \begin{pmatrix}0\\k_7\\0\end{pmatrix},\ L = \begin{pmatrix}1\\0\\0\end{pmatrix}$$
$$\phi(x,u) = sign(Ax(t)),\ A = (0 \quad 1 \quad 0) \tag{34}$$

The problem is to design the virtual sensor for $v(t) = x_2(t)$. Since the system contains a non-smooth nonlinearity, the known methods cannot be used in this case. It can be shown that (35).

$$H_v = (0 \quad 1 \quad 0),\ L_0 = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \tag{35}$$

Equation (19) has a solution with $\lambda = k_2$: $N = (1 \quad 0)$, $J_* = (0 \quad k_3 \quad 0)$, and $\Psi = NL_0 = (0 \quad 1 \quad 0)$. One can check that (8) and (11) are fulfilled, and Equations (7) and (10) with $A^{'} = A$ gives $G_* = 0$, $P_* = k_7$, $A_* = (1 \quad 0 \quad 0)$, $H_v = 1$, $Q = 0$. The linear model is described by the Equation (36).

$$\dot{x}_*(t) = k_2x_*(t) + k_3y_2(t), \quad v(t) = x_*(t) \tag{36}$$

The nonlinear term is given by $P_*\Psi(x_*(t)) = k_7sign(x_*(t))$. The nonlinear model is described by Equation (37).

$$\dot{x}_*(t) = k_2x_*(t) + k_2y_2(t) + k_7sign(x_*(t)), \quad v(t) = x_*(t) \tag{37}$$

since $k_2 < 0$, the model is stable and can be used as a virtual sensor.

Simulation results with $k_1 = k_3 = k_6 = k_7 = 1$ and, $k_2 = k_4 = k_5 = -1$ are given in Fig. 1.

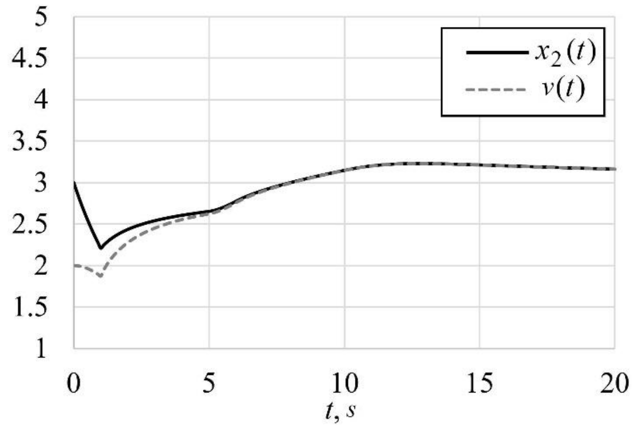

Fig. 1. The behavior of the variables $x_2(t)$ and $v(t)$

6. Conclusion

In this paper, the problem of virtual sensor design has been studied for systems described by nonlinear models with non-smooth nonlinearities under external disturbance. To solve the problem, the logic-dynamic approach has been used. The significance of this approach is that it allows to solve the problem for nonlinear systems by methods of linear algebra and to design the virtual sensor of minimal dimension insensitive to the disturbance. The limitation of the approach is that it is not applicable to non-stationary systems. The problem of virtual sensor location has been solved for the problem of sensor fault identification based on sliding mode observers. New knowledge contributed by the paper is a method to study the system with non-smooth nonlinearities and to achieve the invariance with respect to the disturbance based on the different canonical forms. The future research direction is the virtual sensor design for hybrid nonlinear dynamic systems.

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