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Improved Design of Nonlinear Control Systems with Time Delay

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ABSTRACT

It is well known that time delay in nonlinear control systems may lead to the deterioration or even destabilization of the closed-loop systems. Therefore, specific analysis techniques and design methods are needed to be developed for nonlinear control systems in the presence of time delay. This chapter aims to give a broad overview of the stability and control of nonlinear time-delay systems. Firstly, we present some motivations and a comprehensive survey for the study of time-delay systems. Then, a brief overview of some control approaches is provided, specifically, the Lyapunov-Krasoviskii functional method for high-order polynomial uncertainties nonlinear time-delay systems, and nonlinear time-delay systems with nonlinear input, the Lyapunov-Razumikhin method for triangular structure nonlinear time-delay systems, dynamic gain control for full state time-delay systems. Finally, an application in chemical reactor systems is provided and some related open problems are discussed.

1. Introduction

Time delay is an inherent characteristic of physical systems when materials or energy transmit through a certain route [1]. The phenomenon of time delay exists in various engineering systems such as chemical process, power systems, rolling systems, long transmission lines in pneumatic systems, systems controlled by communication networks, etc. The existence of time delay may lead to deterioration of the closed-loop performance, and even destabilize the systems. Therefore, the stability analysis and control design of time delay systems are significant for practical engineering applications [2]-[6].

Linear systems with time delays have got widely studied and the results are relatively mature [2], [7]-[8]. The early research for time delay systems was mainly based on the classical frequency domain and the transfer function [2]. Through the characteristic analysis of the transfer function roots, at the same time with the aid of Nyquist stability criterion and the small gain theorem, the system stability condition and the basic principles of the controller design are given. In the time domain, based on state space form, the Lyapunov-Krasoviskii method [9]-[12] and Lyapunov-Razumikhin method [8], [13] are generally employed, then the results are often obtained in the form of linear matrix inequalities(LMIs). In general, linear model is an approximation of a real nonlinear system and modeling errors always exist. Early studies of control theory mainly focused on linear system models because the system is relatively simple and high accuracy performance is not required. However, most of the systems encountered in engineering processes are nonlinear in essence [14], in addition, with the rapid development of science and technology, the industrial application systems are becoming much more complex and the accuracy requirement is also increasing. It is not easy to achieve expected control objectives based on linearized models of industrial processes, for instance, the global stability can not be achieved as the linearized model is only locally feasible. Therefore, direct research on practical nonlinear system models should be launched to obtain effective nonlinear controllers. In the survey on time delay systems [4], [15], the authors pointed out that delay systems are still inviting further investigation and with full of challenge.

Compared with linear time delay systems, the analysis and synthesis for nonlinear time-delay systems are more difficult and challenging for the reasons that: (i) It is not easy to select Lyapunov functional for nonlinear time delay systems on account of the specific system structure. (ii) It is difficult to compensate for the time delay effect while designing nonlinear controllers, because the time delay is variable in practical systems and it is impossible to obtain the exact value of time delay.

At present, the study of nonlinear time delay systems is mainly focused on two types: quasi-nonlinear time delay systems and pure-nonlinear time delay systems. The quasi-nonlinear time delay system is the one that the nonlinear time delay term in the system generally satisfies the Lipschitz condition, or its boundary is a first-order linear function. For this kind of systems, the existing methods are mainly the direct translation of the linear time delay systems, namely, select the quadratic Lyapunov-Krasoviskii functional to obtain delay-independent result, or select the quadratic Lyapunov function and then use the Razumikhin lemma to obtain delay-dependent result [16]-[18].

On the other hand, the constraints on nonlinear time-delay terms for pure-nonlinear time delay system are weak or unrestricted, thus this kind of system is a more general nonlinear system. Most of the existing literature consider the pure-nonlinear time delay systems with certain structures and assumptions. For instance, with the nonlinear uncertainties satisfy the high-order polynomial form, Ref. [19]-[20] investigated the robust adaptive control problems based on Lyapunov-Krasoviskii functional and Razumikhin lemma. Ref. [21]-[23] investigated the the adaptive control problem for nonlinear time delay systems with uncertainties that are bounded by smooth nonlinear functions. With dead-zone nonlinear input, Ref. [24] focused on the tracking control problem for a class of nonlinear time delay systems. The systems with triangular structures are also popular, which have attracted a lot of researchers' attention. For nonlinear time delay systems in upper triangular form, based on the forwarding and saturation design method, the adaptive state feedback problem was studied in [25]-[26]. For nonlinear time delay systems in lower triangular form, based on the backstepping design method, the robust control was investigated in [27]-[31]. Specially, the dynamic gain method was proposed to stabilize full state time delay nonlinear systems in [30]-[31], in which there were no growing conditions on smooth nonlinear functions. Ref. [19], [32], [24] are exactly targeted at addressing the control of nonlinear time-delay systems.

2. Control of Nonlinear Time Delay Systems

2.1. Robust Stabilization Against Nonlinear Uncertainties

Consider a class of dynamic systems described by the following differential-difference equations

$$\dot{x}(t) = Ax(t) + Bu(t) + \sum_{j=1}^{r} E_j(x(t - h_j(t)), t) \tag{1a}$$

$$x(t) = \psi(t), \quad t \in [t_0 - \tau, t_0] \tag{1b}$$

where $t \in R$ is the time, $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, A and B are the known constant matrices of appropriate dimensions, $E_j(\cdot) : R^n \times R \to R^n, j \in \{1, 2, \cdots, r\}$, is nonlinear continuous vector function which represent delayed state perturbations for the system. In addition, the time delay $h_j(t), j = 1, 2, \cdots, r$, are assumed to be time-varying. The initial condition is given by (2.1.b) where $\psi(t)$ is a continuous function on $[t_0 - \tau, t_0]$, and $\tau := \max\{h_j(t), j = 1, 2, \cdots r\}$.

In this section we will propose a class of adaptive robust state feedback controller based on Lyapunov-Krasoviskii method. The following standard assumption is needed.

Assumption 1: The pair $\{A, B\}$ given in (1a) is completely controllable.

Assumption 2: There exists the continuous vector function $\eta_j(\cdot) : R^n \times R \to R^m, j \in \{1, 2, \cdots, r\}$ such that for all $(x, t) \in R^n \times R$

$$E_j(x(t - h_j(t)), t) = B\eta_j(x(t - h_j(t)), t) \tag{2}$$

and the following inequalities are satisfied:

$$\|\eta_j(x(t - h_j(t)), t)\| \leq \sum_{i=1}^{s} \beta_{ij} \|x(t - h_j(t))\|^i \tag{3}$$

where $\|\cdot\|$ denotes the Euclidean norm, β_{ij} is an unknown positive constant.

Assumption 3: The derivative of each time varying delay is less than 1, that is $\dot{h}_j(t) \leq \alpha_j < 1$, where α_j is a positive constant.

From assumption 1, there exists any scalar μ and any positive symmetric matrix $Q \in R^{n\times n}$, that the following Riccati equation

$$A^T P + PA - \mu PBB^T P = -Q \tag{4}$$

has a solution $P \in R^{n\times n}$ which is also a symmetric positive definite matrix.

Define the Lyapunov-Krasovskii functional candidate for system (1a),(1b) as follows:

$$W(x, \overline{\theta}) = \sum_{i=1}^{s} \frac{1}{i} \left(x^T P x\right)^i + \sum_{i=1}^{s} \sum_{j=1}^{r} \int_{t-h_j(t)}^{t} F_{ij} \|x(z)\|^{2i} dz + \frac{1}{2k} \widetilde{\theta}^2 \tag{5}$$

where matrix P is the solution of algebraic Riccati differential equation (4), F_{ij} is positive scalar, $\widetilde{\theta} = \theta - \overline{\theta}$, θ is defined as

$$\theta = \sum_{j=1}^{r} \sum_{i=1}^{s} \frac{\beta_{ij}^2}{4(1 - \alpha_j) F_{ij}} \tag{6}$$

Propose the following memoryless robust state feedback controller

$$u(t) = -\frac{\mu}{2} B^T P x - \overline{\theta} B^T \frac{\partial V}{\partial x}^T \tag{7}$$

and $\overline{\theta}$ is the adaptive parameter with adaptive law

$$\frac{d\overline{\theta}(t)}{dt} = k \left\| \frac{\partial V}{\partial x} B \right\|^2 - kl\overline{\theta} \tag{8}$$

Consequently, the time derivative of $W(\cdot)$ along the trajectories of closed-loop system satisfies

$$\frac{dW(x, \overline{\theta})}{dt} < -\sum_{i=1}^{s} \xi_i \|x(t)\|^{2i} + \frac{1}{2} l\theta^2 \tag{9}$$

As we know that θ is a constant and l is an adjustable parameter, it is easy to obtain that the closed loop system is robust uniformly ultimately bounded stable in light of Lyapunov stability theory.

Theorem 1 [19]: Consider the system (1a),(1b) satisfying Assumption 3, then the state feedback controller (7) with adaptive law (8) will render the closed-loop system uniformly ultimately bounded (UUB) stable.

2.2. Robust Control of Triangular Structure Nonlinear Time Delay Systems

Consider the following time delay system

$$\left\{\begin{array}{l} \dot{x}_i(t) = x_{i+1}(t) + F_i(\overline{x}_i(t)) + H_i(\overline{x}_i(t), \overline{x}_i(t - d_i(t)), \delta_i(t)), \\ \qquad i = 1, \cdots, n-1 \\ \dot{x}_n(t) = u(t) + F_n(\overline{x}_n(t)) + H_n(\overline{x}_n(t), \overline{x}_n(t - d_n(t), \delta_n(t))) \end{array}\right. \tag{10}$$

where $x_i \in \Re$ and $u \in \Re$ are the state and the control input of the system, respectively. $d_i(t)$ is the time-varying time delay in x_i-subsystem, which satisfies $d_i(t) \leq \tau$, where τ is a positive scalar, $\delta_i(t)$ is the uncertain time varying parameter. $\overline{x}_i(t) = [x_1(t), x_2(t), \cdots, x_i(t)]^T$. $F_i(\cdot)$ are known smooth nonlinear functions and $H_i(\cdot)$ are unknown uncertain nonlinear functions.

In this part, we will construct a state feedback controller based on Razumikin lemma. For system (10), we impose the following assumption:

Assumption 4: The uncertain nonlinear functions $H_i(\overline{x}_i(t), \overline{x}_i(t - d_i(t)), \delta_i(t))$ yield

$$\begin{aligned} &|H_i(\overline{x}_i(t), \overline{x}_i(t - d_i(t)), \delta_i(t))| \\ &\leq \theta_i \widetilde{\eta}_i(\overline{x}_i(t)) + \sum_{j=1}^{i} \vartheta_{ij} \widetilde{\beta}_{ij}(\|\overline{x}_j(t - d_i(t))\|) + \varepsilon_i, \end{aligned} \tag{11}$$

where ε_i are known positive scalars, θ_i and ϑ_{ij} are unknown positive scalars, $\widetilde{\eta}_i(\cdot)$ are known positive and smooth nonlinear functions, $\widetilde{\beta}_{ij}(\cdot)$ are class-$\mathbb{k}_\infty$ functions and $\widetilde{\eta}_i(0) = \widetilde{\beta}_{ij}(0) = 0$.

For the system (10) we choose the following state transformation

$$\left\{\begin{array}{l} z_1(t) = x_1(t) \\ z_i(t) = x_i(t) - \alpha_{i-1}(\overline{x}_{i-1}(t)), i = 2, 3 \cdots n \end{array}\right. \tag{12}$$

where $\alpha_{i-1}(\cdot)$ are the smooth virtual control inputs with $\alpha_{i-1}(0) = 0$.

Now we revisit the useful Razumikhin lemma [8].

Lemma 1: Suppose $f : \Re \times C \to \Re^n$ takes $\Re\times$(bounded sets of C) into bounded sets of $\Re^n$ and consider the retarded functional differential equation (RFDE)

$$\dot{x}(t) = f(t, x_t).$$

Suppose that $u(s), v(s)$ and $w(s)$ are continuous nondecreasing functions, $u(s) \to \infty$ as $s \to \infty$. If there are a continuous function $V : \Re\times\Re^n \to \Re$, a continuous nondecreasing function $p : \Re^+ \to \Re^+$, $p(s) > s$ for $s > 0$ and a constant $\sigma \geq 0$ such that

(1) $u(\|x\|) \leq V(t, x) \leq v(\|x\|)$

(2) $\dot{V}(t, x(t)) \leq -w(\|x(t)\|) + \sigma$, if $V(t + \theta, x(t + \theta)) < p(V(t, x(t)))\ \forall\theta \in [-\tau, 0]$

then the solutions of the RFDE(f) are uniformly ultimately bounded. In this case, it is said that the system is UUB stable. If $\sigma = 0$, the system is said to be asymptotically stable.

Choose the following quadratic Lyapunov function

$$V = \sum_{j=1}^{n} z_j^2(t). \tag{13}$$

Obviously, condition (1) of lemma 1 is satisfied. Further we choose $p(V(x(t))) = q^2 V$, here q is a positive scalar satisfying $q > 1$. So if the following condition holds for $0 \le d_j(t) \le \tau$

$$\left\| \overline{z}_j (t - d_j(t)) \right\| \le \left\| z(t - d_j(t)) \right\| < q \left\| z(t) \right\|, \tag{14}$$

and if we can design a controller such that condition (2) of lemma 1 is satisfied, the closed-loop system will be UUB stable.

Based on the backstepping method, design the controller as

$$\begin{aligned} u(t) = & -\frac{1}{2} k z_n(t) - \frac{1}{2} h_n z_n(t) - z_{n-1}(t) - \frac{1}{2} a_n z_n(t) - \frac{1}{2} b_n z_n^3(t) \\ & + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \left(x_{j+1} + F_j(\overline{x}_j(t)) \right) - \frac{1}{2} \sum_{j=1}^{n-1} \sum_{l=1}^{j} z_n(t) \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \eta_{jl}(\overline{z}_l(t)) \right)^2 \\ & - \frac{1}{2} \sum_{l=1}^{n} z_n(t) \eta_{nl}^2(\overline{z}_l(t)) - \frac{1}{2} \sum_{j=1}^{n} \sum_{l=1}^{n} n^2 q^2 z_n(t) \overline{\beta}_{nj}^2 (nq |z_l(t)|) - F_n(\overline{x}_n(t)) \end{aligned} \tag{15}$$

where α_{n-1} is the corresponding virtual controller, and finally get

$$\dot{V} \le -kV + \sum_{i=1}^{n} \left((c_i - a_i) z_i^2 - b_i z_i^4 \right) + \sum_{i=1}^{n} \sum_{j=1}^{i} \frac{\varepsilon_j^2}{h_i} \tag{16}$$

If parameter $a_i \ge c_i$, we have

$$\dot{V} \le -kV + \sum_{i=1}^{n} \sum_{j=1}^{i} \frac{\varepsilon_j^2}{h_i}, \tag{17}$$

and if parameter $a_i < c_i$, one has

$$\dot{V} \le -kV + \sum_{i=1}^{n} \frac{(c_i - a_i)^2}{4b_i} + \sum_{i=1}^{n} \sum_{j=1}^{i} \frac{\varepsilon_j^2}{h_i}. \tag{18}$$

From (17) and (18), the resulting closed-loop system is UUB stable based on Lemma 1.

With the above analysis, we have the following main result:

Theorem 2 [32]: For system (10) satisfying Assumption 4, the state feedback controller (15) renders the resulting closed-loop system UUB stable.

2.3. Adaptive tracking controller design

Consider the following time delay systems with unknown dead-zone input

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t), i = 1, 2, \cdots, n-1 \\ \dot{x}_n(t) = f(t, x_1(t - d_1(t)), x_2(t - d_2(t)), \cdots, x_n(t - d_n(t))) + \Gamma(u(t)) \end{cases} \tag{19}$$

$$x(t) = \varphi(t), \ t \in \left[-\overline{d} \ \ 0 \right].$$

where $x_i(t) \in \Re$ and $u(t) \in \Re$ are the state variable and control input of system respectively, $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$, $f(\cdot)$ is the uncertain smooth nonlinear function, $d_i(t)$ are the

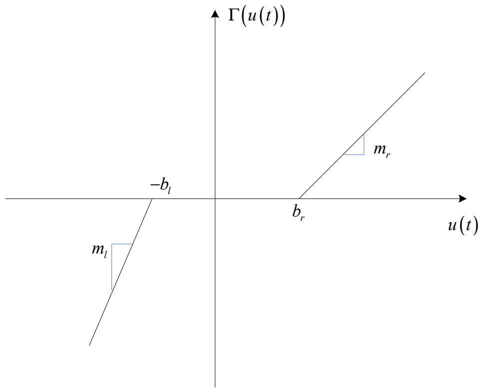

Fig. 1. Dead-zone nonlinearity

delay parameters satisfying $d_i(t) \leq d_i^*$ and $\dot{d}_i(t) \leq \overline{d}_i < 1$, $\Gamma(u(t))$ is a single non-symmetric dead-zone input nonlinearity defined as:

$$\Gamma(u(t)) = \begin{cases} m_r(u(t) - b_r) & \text{if } u(t) \geq b_r \\ 0 & \text{if } -b_l < u(t) < b_r \\ m_l(u(t) + b_l) & \text{if } u(t) \leq -b_l \end{cases}. \tag{20}$$

The non-symmetric dead-zone input is shown in Fig. 1. The parameters m_r and m_l stand for the right and the left slope of the dead-zone characteristic. The parameters b_r and b_l represent the breakpoints of the input nonlinearity.

The following assumptions are imposed on system (19):

Assumption 5: Parameters m_r, m_l, b_l and b_r are positive and unknown.

Assumption 6: The uncertain function $f(\cdot)$ satisfies the following inequality

$$\begin{aligned} &\| f(t, x_1(t - d_1(t)), x_2(t - d_2(t)), \cdots, x_n(t - d_n(t))) \| \\ &\leq \sum_{i=1}^{n} \theta_i \alpha_i(|x_i(t - d_i(t))|) + \gamma, \end{aligned} \tag{21}$$

where θ_i and γ are unknown positive scalars and functions $\alpha_i(\cdot)$ are known smooth class-κ function.

Similarly to [33], the dead-zone input can be expressed in the following form

$$\Gamma(u(t)) = m(t)u(t) + d(t) \tag{22}$$

where

$$\begin{aligned} m(t) &= \begin{cases} m_l & \text{if } u(t) \leq 0 \\ m_r & \text{if } u(t) > 0 \end{cases}, \\ d(t) &= \begin{cases} -m_r b_r & \text{if } u(t) \geq b_r \\ -m(t)u(t) & \text{if } -b_l < u(t) < b_r \\ m_l b_l & \text{if } u(t) \leq -b_l \end{cases}. \end{aligned}$$

Then, one knows that there exists a positive scalar η such that $\eta \leq m_l$ and $\eta \leq m_r$.

Problem: For system (19) with Assumption 5 and 6, given signal

$$x_d(t) = (x_{d1}(t), x_{d2}(t), \cdots, x_{dn}(t))^T = \left(y_d(t), \dot{y}_d(t), \cdots, y_d^{(n-1)}(t)\right)^T,$$

where $y_d(t)$ is sufficiently smooth and $\dot{x}_d(t) \in L_\infty$, design a smooth and memoryless state feedback controller such that the system state $x(t)$ exponentially tracks the signal $x_d(t)$ and the resulting tracking error can be rendered arbitrary small by adjusting design parameters.

We choose the following Lyapunov functional

$$V = V_1 + V_2, \tag{23}$$

with

$$\begin{aligned} V_1 &= \int_0^W \rho(\xi)\, d\xi, \\ V_2 &= \frac{1}{2\eta l_1}\left(\theta^* - \eta\theta(t)\right)^2 + \frac{1}{2\eta l_2}\left(\gamma^* - \eta\gamma(t)\right)^2 \\ &\quad + \sum_{i=1}^{n} \frac{\delta e^{\omega d_i^*}}{1-\bar{d}_i} \int_{t-d_i(t)}^{t} e^{-\omega(t-\xi)} \alpha_i^2\left(2\left|e_i(\xi)\right|\right) d\xi, \end{aligned}$$

where $W = e^T(t)\,Pe(t)$, $e_i(t) = x_i(t) - x_{di}(t)$, θ^* and γ^* are positive scalars, l_1, l_2 are design parameters, $\rho(\cdot)$ is a positive non-decreasing function which yields

$$\delta\left(\sum_{i=1}^{n} \frac{e^{\omega d_i^*}}{1-\bar{d}_i}\alpha_i^2\left(2\left\|e_i(t)\right\|\right) + ce(t)^T e(t)\right) \le \nu\rho\left(e(t)^T Pe(t)\right) e(t)^T Pe(t),$$

in which δ is a positive parameter, c, ω and ν are positive scalars satisfying $\omega \le l_1\sigma_1$, $\omega \le l_2\sigma_2$ and $\nu < \varpi$.

Construct the state feedback controller as

$$u(t) = -\frac{1}{2}\theta(t)\, B^T Pe(t)\,\rho(W) - \frac{1}{2}\gamma(t)\tanh\left(\frac{B^T Pe(t)\,\rho(W)}{\varepsilon}\right), \tag{24}$$

in which

$$\begin{aligned} \dot{\theta}(t) &= l_1\left(B^T Pe(t)\,\rho(W)\right)^2 - l_1\sigma_1\theta(t), \theta(0) > 0, \\ \dot{\gamma}(t) &= l_2 B^T Pe(t)\,\rho(W)\tanh\left(\frac{B^T Pe(t)\,\rho(W)}{\varepsilon}\right) - l_2\sigma_2\gamma(t), \gamma(0) > 0, \end{aligned} \tag{25}$$

where ε, σ_1 and σ_2 are positive scalars.

Theorem 3 [24]: For system (19) with Assumption 5 and 6, with the state feedback controller (24) and the adaptive law (25), the solution of the closed-loop error system exponentially converges to an adjustable region.

2.4. Output feedback control for stochastic delay systems

Consider the following stochastic nonlinear system

$$\begin{aligned} dx_j(t) &= \left(x_{j+1} + f_j(x, x_d, t)\right) dt + g_j^T(x, x_d, t)\, dw \\ y &= x_1;\ j = 1, ..., n \end{aligned} \tag{26}$$

where $x = (x_1, x_2, ..., x_n)^T$ and $x_{n+1} := u$ are state vector and the input of the system, respectively. The unknown positive constant d represents time delay and $w \in R^\kappa$ is an independent standard wiener process defined on a complete probability space. The drift term $f_j(x, x_d, t) : R^{2n+1} \to R$, and the diffusion term $g_j^T(x, x_d, t) : R^{2n+1} \to R^\kappa$ are Borel measurable and satisfy the Assumption 7.

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The control objective of this paper is to design a delay-independent output feedback controller for system (26) such that the output y can track a given reference signal y_r, and all the signals are bounded in probability. Specially, if the $y_r = 0$, the state variables converge to equilibrium almost surely.

Assumption 7: The nonlinear functions $f_j(x, x_d, t)$, $g_j(x, x_d, t)$ are locally Lipschitz in (x, x_d) and $f_j(0, 0, t)$, $g_j(0, 0, t)$ are uniformly bounded in t, satisfying

$$|f_j(x, x_d, t)| \leq \tilde{\varphi}_j(|x_{1d}|) \sum_{k=1}^{j} |x_{kd}| \tag{27}$$

$$|g_j(x, x_d, t)| \leq \tilde{\phi}_j(|x_{1d}|) \sum_{k=1}^{j} |x_{kd}| \tag{28}$$

where the functions $\tilde{\varphi}_j(s) \geq 0$ and $\tilde{\phi}_j(s) \geq 0$ are known smooth non-decreasing functions for $\forall s \geq 0$. Further, there exists a constant $m \geq 0$ and positive real numbers p_f, p_g satisfying

$$\tilde{\varphi}_j(s) \leq p_f + s^m; \tilde{\phi}_j(s) \leq p_g + s^m; \forall s \geq 0. \tag{29}$$

Assumption 8: Both the reference signal $y_r(t)$ and its derivative $\dot{y}_r(t)$ are bounded.

Lemma 2: For any strictly positive real number σ_4, there exist real numbers σ_1 and σ_2, symmetric matrices P_1 and P_2, and column vectors $a = (a_1, a_2, ..., a_n)^T$ and $k = (k_1, k_2, ..., k_n)^T$ satisfying the following set of inequalities:

$$\begin{gathered}
\sigma_1 > 0, \sigma_2 > 0, P_1 > 0, P_2 > 0, \\
P_1\left(A - ac^T\right) + \left(A - ac^T\right)^T P_1 \leq -\sigma_1 I, \\
P_2\left(A - bk^T\right) + \left(A - bk^T\right)^T P_2 \leq -\sigma_1 I, \\
-\sigma_4 P_1 \leq P_1 D + DP_1 \leq \sigma_2 P_1 \\
-\sigma_4 P_2 \leq P_2 D + DP_2 \leq \sigma_2 P_2
\end{gathered}$$

where $c^T = (1, 0, ..., 0)_{1\times n}$, $b^T = (0, ..., 0, 1)_{1\times n}$, $D = diag\{0, 1, ..., n-1\}$ and

$$A = \begin{pmatrix} 0 & & & \\ \vdots & & I_{n-1} & \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

in which I_{n-1} is the $(n-1)$-dimensional identity matrix.

In this section, the dynamic gain observer is constructed first. Further, the controller is designed by the non-recursive method. Design the observer as

$$\dot{\hat{x}}_j = \hat{x}_{j+1} + l^j a_j(y - \hat{x}_1); j = 1, 2, ..., n \tag{30}$$

where $\hat{x}_{n+1} := u$, a_j satisfies Lemma 2 and the dynamic gain l is constructed as

$$\begin{aligned}
\dot{l}(t) = \max\{0; \ & \frac{l}{\sigma_4\sigma_3\lambda^{\varepsilon}_{\min}(P_1)}(-\alpha l + \rho_1(\xi_1, x_1)); \\
& \frac{l}{\sigma_4\lambda^{\varepsilon}_{\min}(P_2)}(-\alpha l + \rho_2(\xi_1, x_1))\Big\}; l(0) = 1
\end{aligned} \tag{31}$$

where α and σ_3 are positive constants. $\varepsilon > 1$ is a constant. σ_4 satisfies $1 - 2\varepsilon m\sigma_4 > 0$. $\rho_1(\xi_1, x_1)$, $\rho_2(\xi_1, x_1)$ are appropriately positive smooth functions.

Select the Lyapunov functional as

$$V = V_e + V_\zeta + \tilde{V} \tag{32}$$

where $V_e = \frac{1}{\varepsilon}\sigma_3\left(\tilde{e}^T P_1 \tilde{e}\right)^\varepsilon$, $V_\zeta = \frac{1}{\varepsilon}\left(\zeta^T P_2 \zeta\right)^\varepsilon$,

$$\begin{aligned}\tilde{V} = \int_{t-d}^{t} e_0^{-\frac{t-s-d}{d}} \Big\{ &\|\zeta_d\|^{2\varepsilon}\left(\bar{\varphi}\left(\xi_1(s), x_1(s)\right)\right. \\ &\left.+\bar{\phi}\left(\xi_1(s), x_1(s)\right) + \mu\left(\xi_1(s)\right) + \bar{\mu}\left(\xi_1(s)\right)\right) \\ &+ \|\tilde{e}_d\|^{2\varepsilon}\left(\bar{\varphi}\left(\xi_1(s), x_1(s)\right) + \bar{\phi}\left(\xi_1(s), x_1(s)\right)\right)\Big\} ds,\end{aligned}$$

$e = x - \hat{x}$, $L_1 = diag\left\{1, l, ..., l^{n-1}\right\}$, $\tilde{e} = L_2^{-1}e$, $L_2 = l^{\sigma_4}L_1$, $\xi_1 = y - y_r$, $\xi_j = \hat{x}_j$, $\zeta = L_2^{-1}\xi$.

Construct the controller as

$$u = -l^n k_1 \xi_1 - l^{n-1} k_2 \xi_2 - ... - l k_n \xi_n \tag{33}$$

where k_j satisfies Lemma 2.

Theorem 4: For the stochastic time delay system (26) satisfying Assumption7–8, we design the observer (30), the dynamic gain (31) and the output feedback controller (33). For the closed-loop system (26), we have following main results: (I) there exists a unique solution on $[-d, \infty)$ for any initial data; (II) the expectation of $\xi_1^{2\varepsilon}$ and the state variables are bounded in probability on $[-d, \infty)$; (III) If the reference signal $y_r = 0$, the state variables converge to equilibrium almost surely.

Remark 1: In this section, we mainly study the full state time delay problem for (26), so we only consider the case that there exist time delay terms in the right-hand of (27)-(28). Of course, Assumption 7 can be easily extended to following form

$$|f_j(x, x_d, t)| \le \tilde{\varphi}_j(|x_{1d}|)\sum_{k=1}^{j}|x_{kd}| + \bar{\varphi}_j(|x_1|)\sum_{k=1}^{j}|x_k|$$

$$|g_j(x, x_d, t)| \le \tilde{\phi}_j(|x_{1d}|)\sum_{k=1}^{j}|x_{kd}| + \bar{\phi}_j(|x_1|)\sum_{k=1}^{j}|x_k|$$

where for $\forall s \ge 0$, $\tilde{\varphi}_j(s) \ge 0$, $\bar{\varphi}_j(s) \ge 0$, $\tilde{\phi}_j(s) \ge 0$ and $\bar{\phi}_j(s) \ge 0$ are known smooth non-decreasing functions and have the same properties with $\tilde{\varphi}_j(s)$ and $\tilde{\phi}_j(s)$ in (29).

3. Application to Chemical Reactor Systems

In chemical industry, the chemical reactor recycle system is very popular. It is well known that a reactor recycle not only increases the overall conversion but also reduces the reaction cost. For the recycling, the input to be recycled must be separated from the yields, then do the separation operation and finally travel through pipes. This set of operations introduce delays in the recycle system. The controller design problem for chemical system has received considerable attentions [29]-[30], [34].

Let us consider a cascade chemical system with two reactors A and B shown in Fig. 2. The compositions C_A, C_B of produce streams from the reactors are the system state, which are to be controlled. A and B are time delay systems themselves, and there exists the time delay on recycling some compositions of A to B. The input of system A comes from the system B and the external

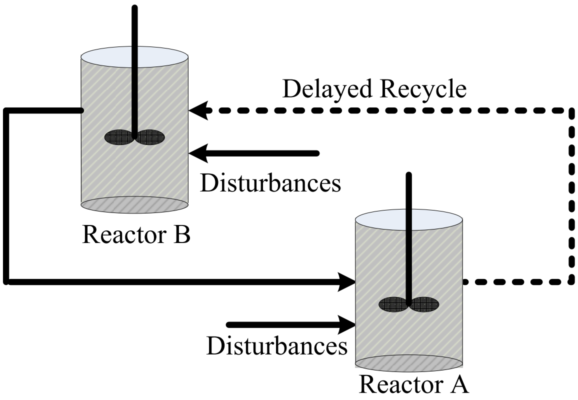

Fig. 2. A cascade chemical reactor system

disturbances, while the input of system B is the delayed system state of A, control input and external disturbances. The whole plant is described by the following model

$$\begin{cases} \dot{C}_A = -k_A C_A - \frac{1}{\theta_A} H_A \left(C_A, C_A \left(t-d_A\right)\right) \\ +\frac{1-R_B}{V_A} C_B + \delta_A \left(t, C_A \left(t-d_A\right)\right) \\ \dot{C}_B = -k_B C_B - \frac{1}{\theta_B} H_B \left(C_B, C_B \left(t-d_B\right)\right) \\ +\frac{R_A}{V_B} C_A \left(t-d_A\right) + \frac{R_B}{V_B} C_B \left(t-d_B\right) \\ +\frac{F}{V_B} u\left(t\right) + \delta_B \left(t, C_B \left(t-d_B\right)\right) \end{cases} \tag{34}$$

where R_i are the recycle flow rates, θ_i are the reactor residence times, k_i are the reaction constants, F is the feed rate, V_i are reactor volumes, H_i are nonlinear functions representing the complex behavior of the systems, δ_i are nonlinear functions for describing the system uncertainties and external disturbances .

With $\delta_A = 0$ and $\delta_B = 0$, we can compute the equilibrium point of the system. Assuming $H_A = C_A + C_A\left(t-d_A\right)$ and $H_B = C_B^2\left(t\right)$, one knows the equilibrium point C_A^* and C_B^* satisfy

$$\left(\frac{2}{\theta_A} + k_A\right) C_A^* = \frac{1-R_B}{V_A} C_B^*$$
$$k_B C_B^* + \frac{1}{\theta_B} C_B^{*2} = \frac{R_A}{V_B} C_A^* + \frac{R_B}{V_B} C_B^*.$$

Further letting $x_1 = C_A\left(t\right) - C_A^*$ and $x_2 = C_B\left(t\right) - C_B^*$ gives

$$\begin{cases} \dot{x}_1 = -k_A x_1 - \frac{1}{\theta_A} x_1 - \frac{1}{\theta_A} x_1 \left(t-d_A\right) \\ +\frac{1-R_B}{V_A} x_2 + \delta_A \left(t, C_A \left(t-d_A\right)\right) \\ \dot{x}_2 = -k_B x_2 - \frac{1}{\theta_B} x_2^2 \left(t\right) + \frac{R_A}{V_B} x_1 \left(t-d_A\right) - \frac{2C_B^*}{\theta_B} x_2 \left(t\right) \\ +\frac{R_B}{V_B} x_2 \left(t-d_B\right) + \frac{F}{V_B} u\left(t\right) + \delta_B \left(t, C_B \left(t-d_A\right)\right) \end{cases}, \tag{35}$$

Obviously, we can see that (35) is a typical nonlinear time delay system.

For system (35), we choose the following parameters

$$\theta_i = 2, k_i = 0.5, R_i = 0.5, V_i = 0.5, F = 0.5.$$

The equilibrium point is $C_A^* = 14/9, C_B^* = 7/3$. Further one has

$$\begin{cases} \dot{x}_1 = -0.5x_1 - 0.5x_1 - 0.5x_1 \left(t-d_A\right) \\ +x_2 + \overline{\delta}_A \left(t, x_1 \left(t-d_A\right)\right) \\ \dot{x}_2 = -0.5x_2 - 0.5x_2^2 \left(t\right) + x_1 \left(t-d_A\right) - \frac{7}{3} x_2 \left(t\right) \\ +x_2 \left(t-d_B\right) + u\left(t\right) + \overline{\delta}_B \left(t, x_2 \left(t-d_B\right)\right) \end{cases} \tag{36}$$

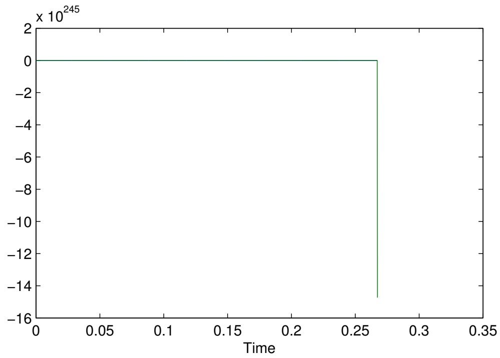

Fig. 3. Response of system state without control

where $\overline{\delta}_A\left(t, x_1\left(t-d_A\right)\right)=\delta_A\left(t, C_A\left(t-d_A\right)\right)$ and $\overline{\delta}_B\left(t, x_2\left(t-d_B\right)\right)=\delta_B\left(t, C_B\left(t-d_B\right)\right).$

The uncertainties are as the following functions: $\delta_A\left(t, x_1\left(t-d_A\right)\right)=0.5 \vartheta_1(t) x_1\left(t-d_A\right)$ and $\delta_B\left(t, x_2\left(t-d_B\right)\right)=0.5 \vartheta_2(t) x_2^2\left(t-d_B\right) e^{0.01 x_2(t-d)}$, where $\left|\vartheta_i(t)\right| \leq 1$.

First, we employ the linear method [34] to design the linear controller. By linearizing the system (2.36) on the zero equilibrium point, The system is expressed as

$$\begin{aligned}\dot{x}(t)= & {\left[\begin{array}{cc}0.5\left(\Delta_1-1\right) & 0 \\ 1 & 1\end{array}\right]\left[\begin{array}{l}x_1\left(t-d_A\right) \\ x_2\left(t-d_B\right)\end{array}\right] } \\ & +\left[\begin{array}{cc}-1 & 1 \\ 0 & -1.3571\end{array}\right] x(t)+\left[\begin{array}{c}0 \\ u(t)\end{array}\right]\end{aligned} \tag{37}$$

where Δ_1 and Δ_2 represent the uncertainties with $\left|\Delta_1\right| \leq 1$ and $\left|\Delta_2\right| \leq 1$.

Based on the linear method, we design $u(t)=K x(t)$ and choose the Lyapunov functional $V=x^T P x+\int_{t-d_A}^t x^T(\xi) Q_1 x(\xi) d \xi+\int_{t-d_B}^t x^T(\xi) Q_2 x(\xi) d \xi$ where P, Q_1 and Q_2 are positive matrices, then by solving the LMI, we have $K=\left[\begin{array}{ll}-0.8886 & -1.3740\end{array}\right].$

Now we illustrate the proposed nonlinear control design procedure [29]. The virtual control input $\alpha_1\left(x_1\right)$ is designed as

$$\alpha_1\left(x_1(t)\right)=1.9165 z_1(t) \tag{38}$$

and the controller is designed as

$$\begin{aligned} u(t)= & -0.5594 x_2+1.9165 x_1+0.5 x_2^2(t)-88.5 z_2(t) \\ & -0.0817 z_2^3(t) e^{0.02 z_2^2(t)}-1.1872 z_2^3 e^{0.073 z_2^2(t)}\end{aligned} \tag{39}$$

We choose the initial values are chosen as $x_1(\xi)=1$ and $x_2(\xi)=-1$ for $\xi \in[-0.25,0]$ at first. The state response is shown in Fig. 3 without control, from which we can see that the system is emanative. With the controller, the simulation results are shown in Fig. 4 (linear controller) and Fig. 5 (nonlinear controller). We can see that the both controllers can render the resulting closed-loop system asymptotically stable. Then we choose the initial values $x_1(\xi)=8$ and $x_2(\xi)=-8$ for $\xi \in[-0.25,0]$. The responses are shown in Fig. 6 (linear controller) and Fig. 7 (nonlinear controller), from which we can see that the nonlinear controller is efficient for the large initial values, while the linear controller is not feasible because of its local stability. The above simulation results further show the effectiveness of the proposed controller design methods.

4. Conclusions

As is well known that nonlinear systems exist in a wide range of real world applications, time delays are also inherent and unavoidable in practice, thus it is important to investigate the stability

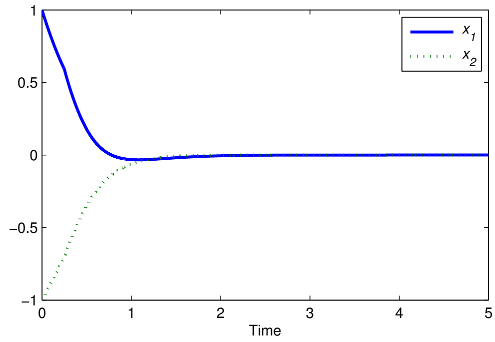

Fig. 4. Response of system state under linear control with small initial vaules

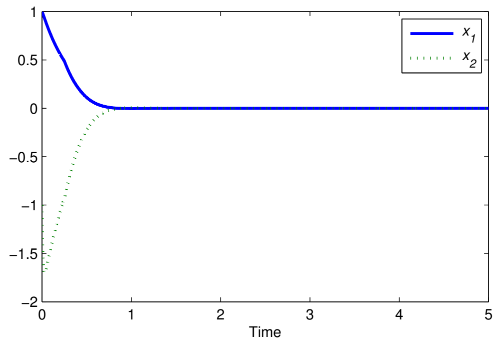

Fig. 5. Response of system state under nonlinear control with small initial values

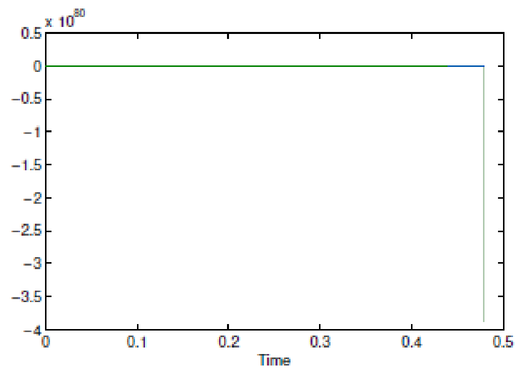

Fig. 6. Response of system state under linear control with large initial values

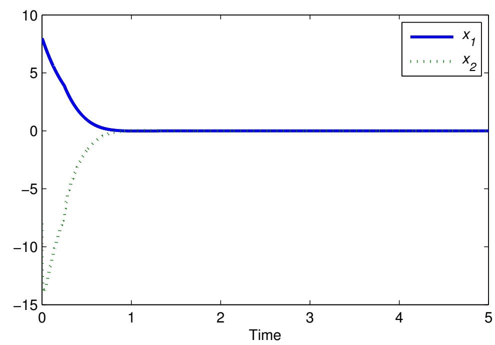

Fig. 7. Response of system state under nonlinear control with large initial values

analysis and control of uncertain nonlinear systems with time delay. This paper has presented a brief overview on the control of nonlinear time delay systems, including especially our initiative work and its application in chemical reactor systems. The problem of robust adaptive feedback control for a class of dynamic systems with multiple delayed state perturbations and high-order polynomial nonlinear uncertainties has been considered by Lyapunov-Krasoviskii functional method. Then, the state feedback control problem for triangular structure nonlinear time delay systems with Razumikhin lemma has been presented. With dead-zone nonlinear input, the tracking control problem for a class of nonlinear time delay systems has been also provided. Finally, combine with dynamic gain technique, output feedback control for full state time delay stochastic nonlinear systems has been investigated without growing conditions.

Although there are many literatures and methods on the control for nonlinear time delay systems at present, there are still some open problems need to be studied. For instance, in the study of nonlinear time delay systems, the time delay is often as a "bad" term to be process, however sometimes it plays the role of a "good" effect. So it is necessary to treatment them separately and give the less conservative results. In addition, with the development of computer technology, the research of discrete systems has attracted more and more attention. Also research should be focus on discrete nonlinear time delay system and this is a meaningful subject. Besides, considering the actual controlled object, such as network control systems, it needs to be further analyzed the specific system mathematical model and applied the theory research results of nonlinear time delay systems to real systems.

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