International Journal of Robotics and Control Systems
Vol. 2, No. 2, 2022, pp. 262-276
ISSN: 2775-2658
http://pubs2.ascee.org/index.php/ijrcs

Coordinated Distributed Voltage Control Methods for Standalone Microgrids

Awatef K. Ali [a,1], Magdi S. Mahmoud [b,2,*]

[a] Electronics Department, National Telecommunications Institute, Nasr City, Cairo-Egypt
[b] Control and Instrumentation Engineering Department, KFUPM, PO Box 5067, Dhahran 3126, Saudi Arabia
[1] 3watef@gmail.com; [2] msmahmoud@kfupm.edu.sa
* Corresponding Author

ARTICLE INFO

Article History
Received February 18, 2022
Revised April 06, 2022
Accepted April 07, 2022

Keywords
Multiagent systems;
Decentralized control;
Distributed generators;
Microgrid;
Distribution network;
Two-level control

ABSTRACT

A microgrid is a small-scale power grid comprising distributed generators (DGs), distributed storage systems, and loads. It will lose contribution from the main grid if it shifts to islanded mode due to pre-planned or unforeseen disturbances. To restore the terminal voltages of all the distributed generators to the reference value, this paper presents three coordinated secondary control strategies. First, motivated by the synchronization control theory of multiagent systems, a distributed control technique is developed where each of the DGs is considered an agent and they exchange information via a communication network. second, a two-level control technique is designed in which a global controller is employed to monitor the overall performance of the DGs by transmitting corrective signals to the local controllers of the agents. In this technique, all the communication is between the global controller and the local controllers without any direct communication between the agents. Third, decentralized control is provided in which each DG is separately controlled by its local controller that operates based on the local feedback measurements. Simulations are carried out on an islanded microgrid consisting of four DGs to illustrate our design approach.

1. Introduction

Integration of DGs into microgrids is one of the most important ways to cope with the ever-growing energy demand and lessen the increasing pressure due to environmental problems around the world. Microgrids integrate a large number of renewable energy sources (such as biomass, solar photovoltaic, wind power, e.t.c), energy storage systems, and local loads to form a small-scale power distribution network. In essence, microgrids can operate in two modes, i.e., the grid-connected and the autonomous/islanded modes. In the grid-connected mode, the microgrid is linked to the utility grid at the point of common coupling by closing the isolation switch. However, when the microgrid goes to the islanded operation by disconnecting from the utility grid due to disturbances, it should not only keep its own frequency and voltage to some reference levels but also transmit the real and reactive power among the DGs and local loads. Another challenge in the distribution networks is the increasing penetration of DGs which results in a number of technical challenges such as frequency deviation, power quality, protection, voltage regulation, and stability. A hierarchical control structure has been proposed for the MG to tackle these challenges [3], [4].

The hierarchical control approach for the microgrid is generally divided into primary, secondary, and tertiary control levels. The primary control is based on the control of each individual DG unit. In other to tackle the problems associated with primary control such as frequency and voltage deviation, secondary control is needed to bring back the frequency and voltage to their reference values. In the tertiary control, the optimal power operation is considered. Nonetheless, in this paper, the secondary control is studied.

The secondary control of the islanded MG has been investigated extensively. In [4]-[8], a centralized control method was proposed, comprising a complex communication architecture for the secondary control. In such methods, a central controller is used to collect the operating information of the entire network and send back control signals to each DG as their local control. In [1], a combined local and centralized control for active and reactive power of PV inverters was investigated. The active and reactive power of PV units is controlled by the rules based on piece-wise linear functions with tunable parameters optimized by the central controller. In [2], a unified controller has been proposed to restore the voltages of DGs integrated into an autonomous microgrid to a certain level. To achieve this, the load flow algorithm was modified to enable sensitivity analysis and its convergence was improved using the Levenberg–Marquardt approach. In [3], a centralized control technique based on a multiperiod optimal power flow algorithm for calculating the reference values of DGs has been proposed to handle the unbalanced operation of the microgrid. In [4], a centralized coordinated model predictive control scheme was designed for a microgrid with high penetration of DGs to stabilize all the bus voltages. In [5], a rule-based model predictive control was used to coordinate the optimal operation of DG units in distributed networks. A dual-stage control has been realized in [6] to suppress voltage fluctuation of DGs. However, centralized control requires complex communication networks, and single-point failures will lead to the failure of the entire communication, which will lessen the reliability of the microgrid. To mitigate these drawbacks, the distributed control approach that requires a sparse communication network could be a better alternative. In this control strategy, there is communication among the local controllers of neighboring DGs, and no central controller is required.

Owing to the flexibility and high efficiency of multi-agent systems, distributed cooperative control of the multi-agent systems has attracted enormous attention in the recent past. In [7], adaptive distributive control has been designed for a time-varying group formation tracking of linear MAS. In [8], a distributed optimal control of linear MAS on digraph has been developed. In [9], a distributed optimal sliding mode control was designed for a linear MAS on directed topologies. The consensus control of MAS without a leader, where the final value of the agents depends on the initial values of the agents has been extensively studied [10], [11], [12], [13]. When all the agents need to follow the desired trajectory, the leader-follower consensus which commands the agents follow its motion is implemented [14], [15], [16], [17], [18]. The synchronization problem for multi-agent systems with nonlinear dynamics has been presented in [19], [20]. The synchronization problem in the case of linear dynamics is studied in [21], [22].

Recently, the authors in [23] developed a distributed discrete-time secondary control for optimal power-sharing of multiple DG distributed networks. In [24], an event-triggered cooperative secondary control for voltage restoration of DGs is presented. In this method, the data transmission burden is generally reduced. In [25], a distributed finite-time consensus control of DGs is presented to restore the voltages of the DGs to a certain reference value. In [26], a leader-follower-based distributed secondary control of multiple DGs are discussed. In [27], the authors addressed accurate power allocation and distributed secondary control of DGs. In [28], a stochastic distributed secondary control scheme is proposed to achieve synchronization of the voltages of the DGs.

It should be noted that both centralized and distributed control schemes require communication networks [29]. When the communication link between any neighbors is failed, it can bring the whole

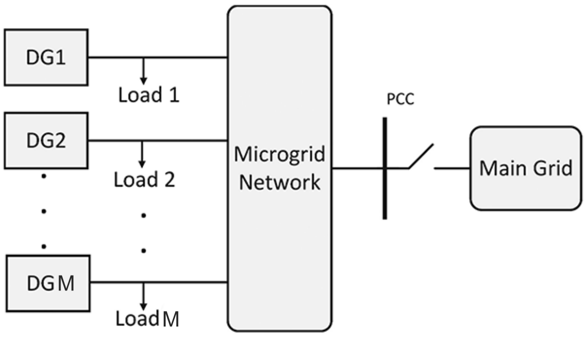

Fig. 1. Schematic diagram of an islanded microgrid with M DGs and M loca loads.

system down. In addition, excessive communication delays can also deteriorate the system's performance. In [30], a decentralized coordination control of DGs is proposed to minimize renewable power curtailment. In [31], decentralized resilient control of an islanded microgrid was presented. In [32], the authors implemented an optimal decentralized control for active/reactive power regulation, minimizing power losses, and maintaining the voltages of the DGs within certain limits. In [33], the authors designed a decentralized control for a distribution network to maintain the microgrid voltage and share active power with DGs. In [34], decentralized optimization is formulated based on mixed-integer programming to minimize power fluctuation in a distributed network. In [35], the authors investigated the impact of increasing decentralized generation on the reliability of a distributed network. Integrating a significant number of DGs can seriously reduce the reliability of the network. The authors concluded that by implementing effective power control schemes, the reliability of the network can be improved.

In this work, we are developing control methods for secondary control of multiple distributed generators (DGs) connected to a standalone microgrid. Specifically, three techniques were addressed:

1. **Distributed control technique** in which the local controller of each DG communicates with their neighbors based on a multiagent system to achieve the voltage synchronization.
2. **Two-level control technique** in which the local controllers operate independently but coordinate accordingly with a central controller to supervise their performance toward restoring the voltage of each DG to a certain reference value.
3. **Decentralized control technique** in which the local controllers have maximum freedom so that each DG is controlled by its own local controller only without any central coordinator.

2. Preliminaries and problem formulation

2.1. Algebraic graph

A directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{A})$ is composed of a set of nodes, $\mathbf{V} = \{\mathbf{v_1}, \ldots, \mathbf{v_M}\}$, a set of edges, $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$, and an adjacency matrix $\mathbf{A} = [\mathbf{a_{ij}}]$ with weights $\mathbf{a_{ij}} = 1$ if $(\mathbf{v_i}, \ \mathbf{v_j}) \in E$ and $\mathbf{a_{ij}} = 0$ otherwise. Define the diagonal matrix $\mathbf{B} = diag(\mathbf{b_1}, \ldots, \mathbf{b_M})$ as the interconnection between followers and the leader (reference). $\mathbf{b_i} > 0$ for $i = 1, 2, \ldots, M$ if an agent i is receiving information from the leader $(\mathbf{v_0})$ and $\mathbf{a_i} = 0$ otherwise. The laplacian matrix $\mathbf{L} = [\mathbf{l_{ij}}]$ of the graph $\mathbf{G}$ is define by $\mathbf{l_{ij}} = -\mathbf{a_{ij}}, \ i \neq j$ and $\mathbf{l_{ij}} = \sum_{j=1, j\neq i}^{M} \mathbf{a_{ij}}, \ i \neq j$.

2.2. Dynamic model of the islanded microgrid

Fig. 1 depicts the schematic diagram of an islanded microgrid with M DGs. A local load is connected to each of the DGs and integrated by an MG network model. Hence, the model of the islanded microgrid is usually composed of the DG model and microgrid network model. The dynamic model of an ith DG is derived based on the following assumption [36], [25]:

1. Power losses in transmission lines are negligible.
2. The inverter dynamics is neglected since it is much faster than its controller.
3. The DGs are receiving sufficient power from their DC sources.

Then the nonlinear state-space model of each of the DG in this study is given by:

$$\begin{bmatrix} \dot{V}_i \\ \ddot{V}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_i \\ \dot{V}_i \end{bmatrix} + \begin{bmatrix} 0 \\ f_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/(\tau_{Q_i} k_{V_i}) \end{bmatrix} u_i^V \tag{1}$$

$$\begin{aligned} f_{1i}(V_i, V_k) = & -\frac{\tau_{Q_i} + k_{V_i}}{\tau_{Q_i} k_{V_i}} \dot{V}_i - \frac{k_{Q_i}\left(Q_{1i} + \sum_{k \in N_i} |B_{ik}|\right)}{\tau_{Q_i} k_{V_i}} V_i^2 \\ & + \frac{k_{Q_i}}{\tau_{Q_i} k_{V_i}} \sum_{k \in N_i} |B_{ik}| V_i V_k \cos(\delta_i - \delta_k) \\ & - \frac{1 + k_{Q_i} Q_{2i}}{\tau_{Q_i} k_{V_i}} V_i - \frac{k_{Q_i}\left(Q_{3i} - Q_i^d\right) - V^d}{\tau_{Q_i} k_{V_i}}. \end{aligned} \tag{2}$$

where V_i is the terminal voltage of each DG, V_k is the voltage of the neighboring DGs, K_{Q_i} is the droop control gain, K_{V_i} is the voltage control gain, Q_i is the reactive power, and B_{ik} is the conductance of the lines between any two DGs.

Equation (1) can be linearized by the following controller.

$$u_i^V = \tau_{Q_i} k_{V_i} [-f_i + u_i] \tag{3}$$

where u_i is the stabilising input of each DG. The linear state-space model of the ith DG is thus:

$$\begin{bmatrix} \dot{V}_i \\ \ddot{V}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_i \\ \dot{V}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \tag{4}$$

Equation (4) can be rewritten as:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i, \quad i = 1, 2, \ldots, M \\ y_i &= C_i x_i \end{aligned} \tag{5}$$

where

$$x_i = \begin{bmatrix} V_i \\ \dot{V}_i \end{bmatrix}; \; A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \; B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \; C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

3. Distributed Control

In this section, we investigate a type of MAS in which communication between control system components, such as actuators, controllers, and sensors, takes place via a shared channel or across a communication network. An output feedback law is developed for the distributed MAS to achieve the leader-following consensus e.g $\lim_{x\to\infty} \|x_i - x_{M+1}\| =, \forall\, i = 1, 2, \ldots, M$. Moreover, $u_{M+1} = 0$. The local neighbor-hood tracking error for is defined as

$$z_i = \sum_{j=1}^{M+1} a_{ij}(x_i - x_j), \quad i = 1, 2, \ldots, M \tag{6}$$

while the global tracking error is defined as

$$z = (\mathcal{L} \otimes \mathcal{I}_p)(x - \bar{1} \otimes x_{M+1}) \tag{7}$$

where $z = [z_1^T, z_2^T, \ldots, z_M^T]^T$. Recall that $u_{M+1} = 0$, then the time derivative of (7) is

$$\dot{z} = (I_M \otimes A)z + (\mathcal{L} \otimes B)U \tag{8}$$

where $U = [u_1^T, u_2^T, \ldots, u_M^T]^T$. It is worth noting that $\lim_{x\to\infty} \|x_i - x_{M+1}\| =, \forall\, i = 1, 2, \ldots, M$, can be realised if and only if (8) is asymptotically stable. The following distributed control law is designed:

$$u_i = C_i \mathcal{K} z_i, \quad i = 1, 2, \ldots, M \tag{9}$$

where K is a feedback gain. In the subsequent section

Theorem 1. *Let $Q = Q^T \geq 0$, $R = R^T > 0$, $c > 0$, and the controller be given as (9). If there exists a positive matrix P satisfying*

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{10}$$

and the feedback gain matrix is defied as $K = R^{-1}B^T P$, the system (8) is asymptotically with respect to the performance index

$$J = \int_0^\infty \frac{1}{2}(z^T \bar{Q} z + U^T \bar{R} U)dt \tag{11}$$

where

$$\begin{aligned} \bar{R} &= c^{-1}\mathcal{L} \otimes R \\ \bar{Q} &= I_M \otimes (Q - PBR^{-1}B^T P) + c\mathcal{L} \otimes PBR^{-1}B^T P \end{aligned} \tag{12}$$

Proof. Define a candidate Lyapunov function as follows:

$$L(z) = z^T (I_M \otimes P) z \tag{13}$$

The time derivative of $L(z)$ gives

$$\begin{aligned} \dot{L}(z) = &\frac{1}{2} z^T (I_M \otimes (PBR^{-1}B^T P - Q))z \\ &+ \frac{1}{2} z^T (I_M \otimes (A^T P + PA))z \\ &- z^T (c\mathcal{L} \otimes PBR^{-1}B^T P)z \end{aligned} \tag{14}$$

Substituting the expression of $\bar{Q}$ into (14) yields:

$$\dot{L}(z) = -\frac{1}{2}(\bar{Q} + c\mathcal{L} \otimes PBR^{-1}B^T P) \tag{15}$$

Therefore, the system (8) is asymptotically stable since. $Q = Q^T \geq 0$, $R = R^T > 0$, $c > 0$. The conditions that guarantee $\bar{Q} \geq 0$ can be proved as follows:

Let $\zeta(z, U) = \frac{1}{2}(U^T + z^T(I_M \otimes cK^T))\bar{R}(U + (I_M \otimes cK)z)$. By using the expressions of K, $\bar{Q}$ and $\bar{R}$ that were defined before, one gets

$$\begin{aligned}\zeta(z, U) &= \frac{1}{2}U^T\bar{R}U + \frac{1}{2}z^T(c\mathcal{L} \otimes PBR^{-1}B^T P)z \\ &+ z^T(\mathcal{L} \otimes PB)U \\ &= \frac{1}{2}U^T\bar{R}U + \frac{1}{2}z^T\bar{Q}z + z^T(\mathcal{L} \otimes PB)U \\ &+ \frac{1}{2}(I_M \otimes (PBR^{-1}B^T P - Q))z\end{aligned} \tag{16}$$

Substituting (10) into (16) yields:

$$\begin{aligned}\zeta(z, U) &= \frac{1}{2}U^T\bar{R}U + \frac{1}{2}z^T\bar{Q}z + z^T(\mathcal{L} \otimes PB)U \\ &+ \frac{1}{2}z^T(I_M \otimes (A^T P + PA))z\end{aligned}$$

$$\begin{aligned}&= \frac{1}{2}U^T\bar{R}U + \frac{1}{2}z^T\bar{Q}z + z^T(\mathcal{L} \otimes PB)U \\ &+ z^T(I_M \otimes (PA))z\end{aligned}$$

$$\begin{aligned}&= \frac{1}{2}U^T\bar{R}U + \frac{1}{2}z^T\bar{Q}z \\ &+ z^T(I_M \otimes P)((I_M \otimes A)z + (\mathcal{L} \otimes B)U)\end{aligned} \tag{17}$$

Using the tracking error in (8), (17) can be expressed as

$$\begin{aligned}\zeta(z, U) &= \frac{1}{2}U^T\bar{R}U + \frac{1}{2}z^T\bar{Q}z + z^T(I_M \otimes P)\dot{z} \\ &= \frac{1}{2}U^T\bar{R}U + \frac{1}{2}z^T\bar{Q}z + \dot{L}(z, U)\end{aligned} \tag{18}$$

From (11), we have

$$\begin{aligned}J &= \int_0^\infty \frac{1}{2}(z^T\bar{Q}z + U^T\bar{R}U)dt \\ &= \int_0^\infty \zeta(z, U)dt - \int_0^\infty \dot{L}(z, U)dt \\ &= \int_0^\infty \zeta(z, U)dt + L(0) - \lim_{t\to\infty}(L(z))\end{aligned} \tag{19}$$

where $L(0)$ is the initial value of $L(z)$. It is worth noticing that $\zeta(z, U) \geq 0$ always holds, and $\zeta(z, U) = 0$ can be achieved by the control law in (9). Hence, it can be deduced from (19) that the minimum value of J is obtained as:

$$J = L(0) - \lim_{t \to \infty} (L(z)) \tag{20}$$

The value of the coupling coefficient c and the condition that $\bar{Q} \geq 0$ can be derived as follows: From (15), and noting that $Q \geq 0$, we can have that

$$\bar{Q} \geq (c\mathcal{L} - I_M) \otimes PBR^{-1}B^T P \tag{21}$$

Hence, $\bar{Q} \geq 0$ can be guaranteed by the condition $c \geq \frac{1}{\sigma_{min}(\mathcal{L})}$. □

4. Decentralised Two-level Control

In this section, a two-level control configuration is used to achieve the secondary voltage coordination control of the DGs instead of agent-to-agent communication. The secondary voltage coordination is accomplished by noticing the group performance and sending the same signal to all the agents.

The two-level control structure comprises a global controller and a local controller. The local optimal control action of each agent is measured by the local controllers. The global controller observes the group performance and provides proper compensatory signals to maintain the global behavior.

The system (5) can be written as an interconnection of M subsystems as follows:

$$\dot{x}_i = A_i x_i + B_i u_i + \sum_{j=1}^{M} \Pi_{ij} x_j, \quad i = 1, 2, \ldots, M \tag{22}$$

where $\sum_{j=1}^{M} \Pi_{ij} x_j$ is the interconnection vector from other susbsystems x_j.

4.1. Local Controller

It is assumed that each subsystem is aiming to find an optimal local controller (u_i^l) which minimises a relevant quadratic cost function

$$J_i = \int_{t_o}^{\infty} \{\|x_i(t_o)\|_{Q_i}^2 + \ u_i^l(t_o)\|_{R_i}^2\} dt \tag{23}$$

Assume $\Pi_{ij} = 0, \ i, j = 1, 2, \ldots, M$ and the system (22) reduces to

$$\dot{x}_i = A_i x_i + B_i u_i, \quad i = 1, 2, \ldots, M \tag{24}$$

The optimum control law for minimizing (23) subject to the constraints given by (24) can be written as

$$u_i = -\hat{K}_i x_i = -\hat{R}_i^{-1} B_i^T \hat{P}_i x_i \tag{25}$$

where $\hat{P}_i$ is the solution of the local algebraic Riccati equation

$$\hat{P}_i A_i + A_i^T \hat{P}_i - \hat{P}_i B_i \hat{R}_i^{-1} B_i^T \hat{P}_i + \hat{Q}_i = 0 \tag{26}$$

The closed-loop decoupled subsystem

$$\dot{x}_i = (A_i - B_i \hat{R}_i^{-1} B_i^T \hat{P}) x_i \tag{27}$$

It has the property that $x_i(t) = x_i(0)\ exp(\alpha t) \longleftarrow 0$ as $t \longleftarrow \infty$.

4.2. Global Controller

The global (central) controller u^g gives corrective controls to counter the interactions between the subsystems. The control input to each subsystem comprises of two components

$$u_i = u_i^l + u_i^g \tag{28}$$

where

$$u_i^g = -\sum_{j=1}^{M} H_{ij} \tag{29}$$

where H_{ij} is a gain matrix for the feedback signals from the jth subsystem to the ith subsystem. Substituting (28) into (22) gives:

$$\dot{x}_i = (A_i - B_i \hat{K}_i) x_i + \sum_{j=1}^{M} [\Pi_{ij} - B_i H_{ij}] x_j \tag{30}$$

Assuming that Π_{ij} are determined through the relation

$$B\Pi = \mathcal{B}^p K \tag{31}$$

where $\mathcal{B}^p$ is the required perturbation in B, $\hat{K} = diag[\hat{K}_1\ \hat{K}_2 \ldots \hat{K}_M]$, and $\Pi = \Pi_{ij}\ i,j = 1,2,\ldots,M$. Then (30) may be written as

$$\dot{x} = (A + T) x_i - (B + \mathcal{B}^p) K x \tag{32}$$

where $A = diag[A_1\ A_2\ \ldots A_M]$, $B = diag[B_1\ B_2\ \ldots B_M]$. It is worth mentioning that $-\mathcal{B}^p$ involves the states of the other subsystems. The objective now is finding $\mathcal{B}^p$ to result in bounded performance without changing K.

Theorem 2. *Let $W \in \mathcal{R}^{m \times m}$ be a skew solution of the matrix equation*

$$W(A + \Pi) + (A + \Pi)^T W + \Pi^T \hat{P} A - A^T \hat{P} \Pi = 0 \tag{33}$$

and the matrix $\underline{P} \in \mathcal{R}^{m \times m}$ given by

$$\underline{P} = (W - \hat{P}\Pi)(A + \Pi)^{-1} \tag{34}$$

be such that $(\underline{P} + \hat{P})$ is positive definite. Then $\mathcal{B}^p$ defined by

$$\mathcal{B}^p = -(\underline{P} + \hat{P})^{-1} \underline{P} B \tag{35}$$

leads to the cont function $\hat{J} = x^T(\underline{P} + \hat{P})x$ for the overall system (32). Moreover, a bound on the suboptimality index $\varepsilon = (\hat{J} - J^*)/J^*$ is given by

$$\varepsilon = \frac{\hat{J} - J^*}{J^*} \tag{36}$$

5. Decentralised Controller

In this section, instead of using local controllers and a global compensatory controller to steer the system (22) to global performance, the objective in this sequel is to solve the DGs control problem via decentralized controllers with sufficient conditions to guarantee the stability of the overall system.

The system (22) can be described as:

$$\dot{x} = Ax + Bu + \Pi' \tag{37}$$

where $\Pi' = \Pi'' x$

$$\Pi'' = \begin{bmatrix} 0 & A_{12} & A_{13} & \cdots A_{1M} \\ A_{21} & 0 & & A_{2M} \\ \vdots & & & \vdots \\ A_{M1} & & & 0 \end{bmatrix}$$

The system (37) can be rewritten as

$$\dot{x} = \tilde{A}x + Bu \tag{38}$$

where

$$\tilde{A} = A + \Pi'' = \begin{bmatrix} A_1 & A_{12} & A_{13} & \cdots A_{1M} \\ A_{21} & A_2 & & A_{2M} \\ \vdots & & \cdots & \vdots \\ A_{M1} & A_{M2} & \cdots & A_M \end{bmatrix}$$

The subsystems of system (37) can be decoupled by assuming the interaction vectors Π_i are equal to zero

$$\dot{x}_i = A_i x_i + B_i u_i, \quad i = 1, 2, \dots, M \tag{39}$$

Each subsystem can be exponentially stabilised with degree γ and minimizes their associated cost function $\tilde{J}_i$ of the form

$$\tilde{J}_i = \int_0^{\infty} e^{\gamma t}(x_i \tilde{Q}_i x_i + u_i \tilde{R}_i u_i)dt \tag{40}$$

with an optimum control law of the form

$$u_i = -\tilde{K}_i x_i = -\tilde{R}_i^{-1} B_i^T \tilde{P}_i x_i \tag{41}$$

where $\tilde{P}_i$ is the solution of the local algebraic Riccati equation

$$\tilde{P}_i A_i + A_i^T \tilde{P}_i - \tilde{P}_i B_i \tilde{R}_i^{-1} B_i^T \tilde{P}_i + \gamma \tilde{P}_i + \tilde{Q}_i = 0 \tag{42}$$

The closed-loop decoupled subsystems are given by:

$$\dot{x}_i = (A_i - B_i \tilde{R}_i^{-1} B_i^T \tilde{P}_i) x_i \tag{43}$$

It is worth noticing that (43) has the property that $x_i(t)e^{(\gamma t)} \longrightarrow 0$ as $t \longrightarrow \infty$. Based on (41), the closed-loop form of system (37) is described by:

$$\begin{aligned} \dot{x} &= (A - B\tilde{R}^{-1}B^T\tilde{P})x + \Pi' \\ \Pi' &= \Pi'' x \end{aligned} \tag{44}$$

where $\tilde{R}^{-1} = diag(\tilde{R}_i^{-1})$, $\tilde{P} = diag(\tilde{P}_i)$ and the solution is

$$\tilde{P}A + A^T\tilde{P} - 2\tilde{P}B\tilde{R}^{-1}B^T\tilde{P} + \gamma\tilde{P} + \tilde{Q} = 0 \tag{45}$$

The interconnection vector Π' will disrupt the stability of the system, and it is requisite to determine sufficient conditions to ensure the stability of the overall system.

Theorem 3. *The overall system (37) can be exponentially stabilised with degree γ in decentralised a form by the control law*

Proof. Choose a Lyapunov function candidate for the overall system (37) as

$$\tilde{L}(x) = x\tilde{Q}x \tag{46}$$

Taking the time derivative of $\tilde{L}(x)$ along (44) yields:

$$\begin{aligned}\dot{\tilde{L}}(x) &= x^T(A^T\tilde{P} - \tilde{P}B\tilde{R}^{-1}B^T\tilde{P})x + \Pi'^T\tilde{P}x \\ &+ x^T(\tilde{P}A - \tilde{P}B\tilde{R}^{-1}B^T\tilde{P})x + x^T\tilde{P}\Pi'\end{aligned} \tag{47}$$

Using (44), then (47) will be

$$\begin{aligned}\dot{\tilde{L}}(x) &= x^T(\tilde{P}A + A^T\tilde{P} - \tilde{P}B\tilde{R}^{-1}B^T\tilde{P})x \\ &+ x^T\Pi''^T\tilde{P}x + x^T\tilde{P}\Pi''x\end{aligned} \tag{48}$$

By substituting (45) into (48), we get

$$\dot{\tilde{L}}(x) = -x^T\Theta x \tag{49}$$

where

$$\begin{aligned}\Theta &= \gamma\tilde{P} + G - (\tilde{P}\Pi'' + \Pi''^T\tilde{P}) \\ G &= \tilde{Q} + \tilde{P}B\tilde{R}^{-1}B^T\tilde{P}\end{aligned}$$

$\dot{\tilde{L}}(x)$ should be negative definite for (44) to be stable. Then the matrix Θ should be positive definite such that

$$\gamma\sigma_{min}(K_i) + \sigma_{min}(G_i) > 2\|\Pi''\|\sigma_{max}(K_i) \tag{50}$$

□

6. Simulation Results

In this section, numerical simulations are conducted to verify the performance of the presented control techniques to achieve the secondary control of the terminal voltages of the DGs integrated into an islanded microgrid. The parameter of the DGs and the microgrid are obtained from [36]. The reference voltage is set as 320 $volts$. The communication topology of the DGs under distributed control is shown in Fig. 2.

The simulation results are presented in Fig. 3 and Fig. 4. The synchronization of the terminal voltages of the DGs under distributed control is depicted in Fig. 3. The convergence of the terminal voltages of the Dgs under two-level control is shown in Fig. 4. The performance of the decentralised controller is shown in Fig. 5.

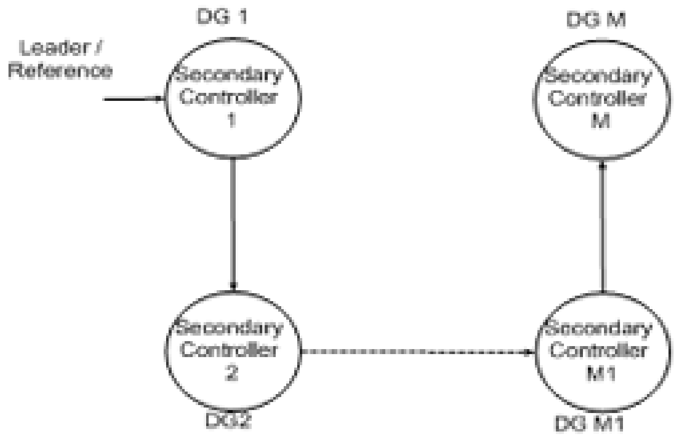

Fig. 2. Communication topology of the DGs under distributed control

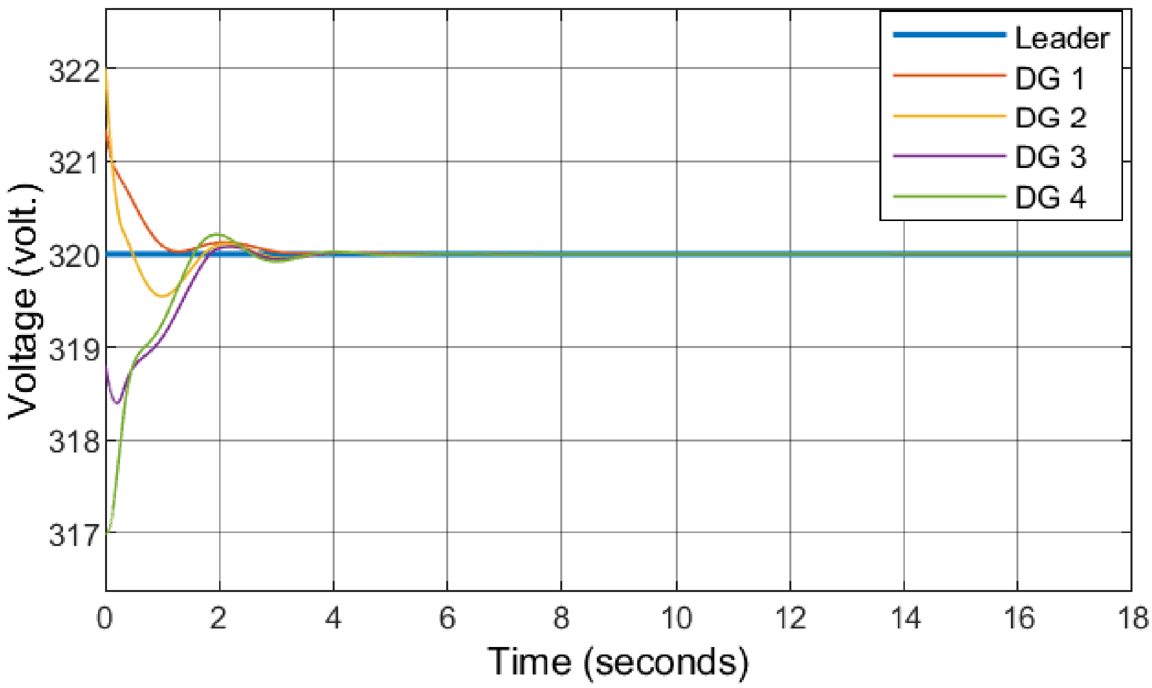

Fig. 3. Voltage responses of the DGs under distributed control.

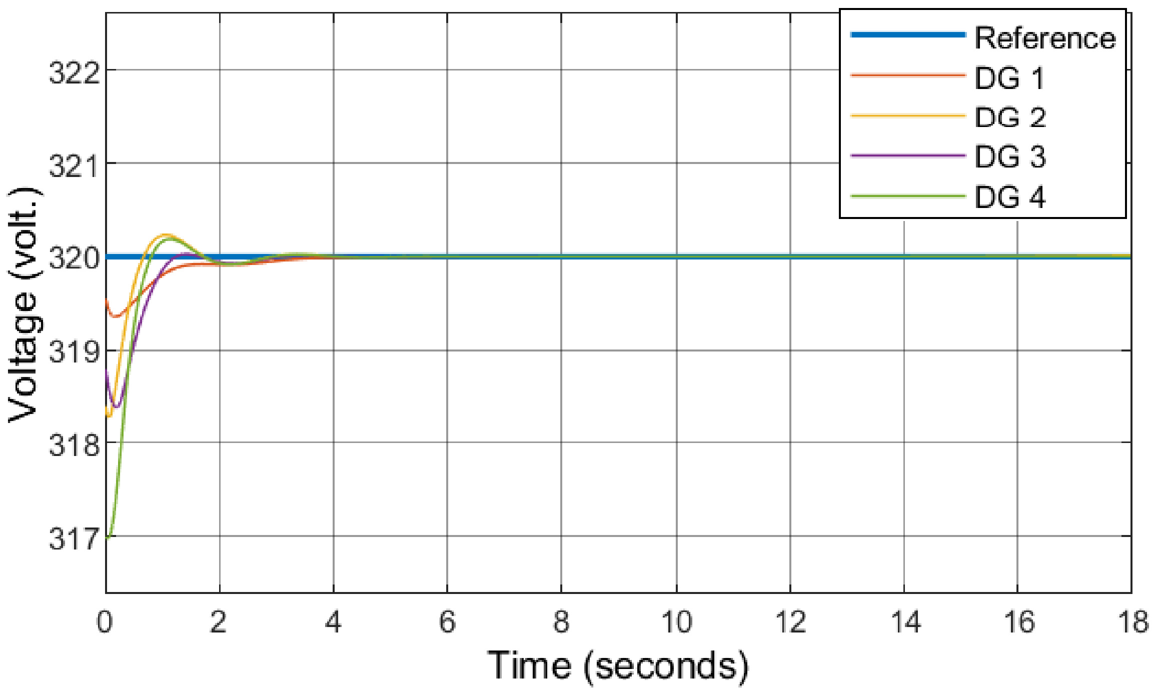

Fig. 4. Voltage responses of the DGs under two-level control.

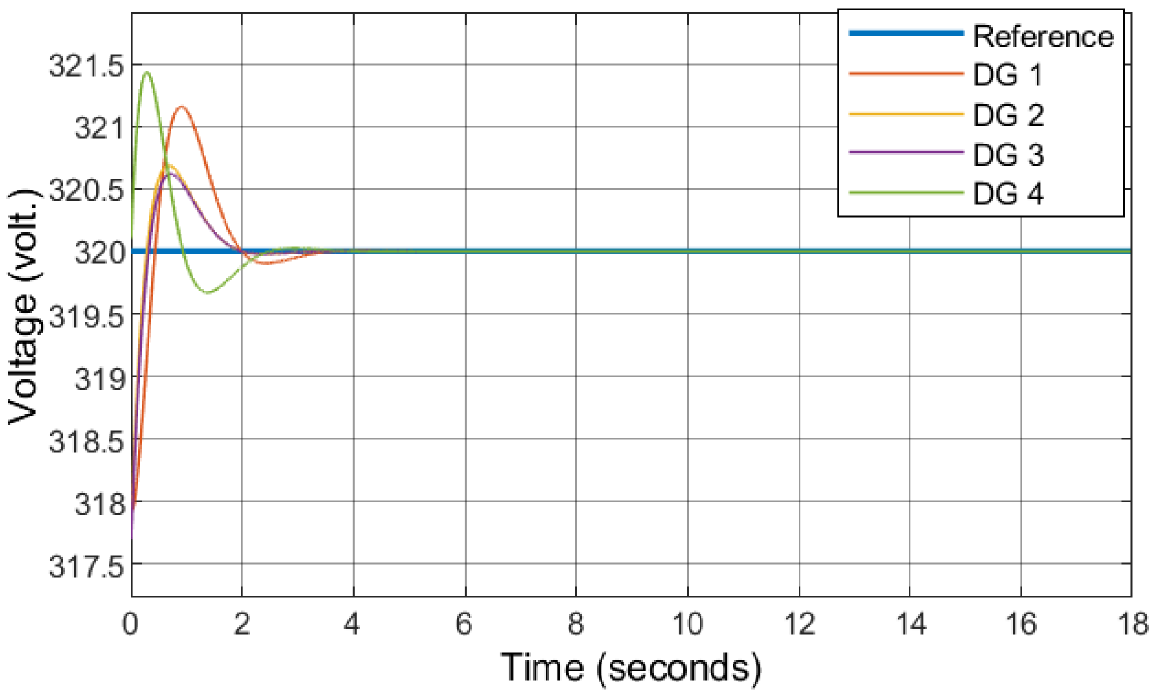

Fig. 5. Voltage responses of the DGs under decentralised control.

7. Conclusions

In this paper, three different approaches for secondary control of DGs integrated into an islanded microgrid were presented. First, a distributed control method was investigated. The controller here was developed using the synchronization control theory of multi-agent systems to derive the terminal voltages of the DGs to follow the leader. Second, a two-level control approach was formulated. This controller consists of local controllers for regulatory actions and a global controller to obtain the desired voltage coordination. Third, a decentralized control was designed to stabilize each DG unit independently and restore the terminal voltages to the reference value without any communication between the local controllers. Numerical simulations were provided to demonstrate the performance of the controllers. Future work will consider cloud-based control for DGs so that the voltage coordination among them will be achieved by communicating with a shared warehouse.

Author Contribution: All authors contributed equally to the main contributor to this paper. All authors read and approved the final paper.
Funding: This research received no external funding.
Conflicts of Interest: The authors declare no conflict of interest.

References

[1] S. Weckx, C. Gonzalez and J. Driesen, "Combined central and local active and reactive power control of pv inverters," *IEEE Trans. Sustain. Energy*, vol. 5, no. 3, p. 776-784, 2014, https://doi.org/10.1109/TSTE.2014.2300934.

[2] E. A. Bruno, "Centralised secondary control for islanded microgrids," *IET Renewable Power Generation*, vol. 14, pp. 1502-1511, 2020, https://doi.org/10.1049/iet-rpg.2019.0731.

[3] S. Karagiannopoulos, P. Aristidou, and G. Hug, "A centralised control method for tackling unbalances in active distribution grids," *2018 Power Systems Computation Conference (PSCC)*, 2018, pp. 1-7, https://doi.org/10.23919/PSCC.2018.8442493.

[4] G. Yifei, W. Qiuwei, G. Houlei, and C. Xinyu, "Mpc-based coordinated voltage requlation for distribution networks with distributed generation and energy storage system," *IEEE Trans. on Sustainable Energy*, vol. 10, no. 4, pp. 1731-1739, 2020, https://doi.org/10.1109/TSTE.2018.2869932.

[5] A. Dutta, S. Ganguly, and C. Kumar, "Coordinated voltage control of active distribution networks in presence of pv and energy storage system," *2021 IEEE 12th Energy Conversion Congress Exposition Asia (ECCE-Asia)*, 2021, pp. 2028-2033, https://doi.org/10.1109/ECCE-Asia49820.2021.9479012.

[6] A. Dutta, S. Ganguly and C. Kumar, "Coordinated volt/var control of pv and ev interfaced active distribution networks based on dual-stage model predictive control," *IEEE Systems Journal*, 2021, https://doi.org/10.1109/JSYST.2021.3110509.

[7] J. Hu, P. Bhowmick, and A. Lanzon, "Distributed adaptive time-varying group formation tracking for multiagent systems with multiple leaders on directed graphs," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 1, pp. 140-150, 2020, https://doi.org/10.1109/TCNS.2019.2913619.

[8] Z. Zhang, W. Yan, and H. Li, "Distributed optimal control for linear multiagent systems on general digraphs," *IEEE Transactions on Automatic Control*, vol. 66, no. 1, pp. 322-328, 2021, https://doi.org/10.1109/TAC.2020.2974424.

[9] Z. Zhang, H. Li, and W. Yan, "Fully distributed control of linear systems with optimal cost on directed topologies," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 68, no. 1, pp. 336-340, 2021, https://doi.org/10.1109/TCSII.2020.2987610.

[10] Wei Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655-661, 2005, https://doi.org/10.1109/TAC.2005.846556.

[11] H. A trianfar and M. Haeri, "Average consensus in networks of dynamic multi-agents with switching topology: Infinite matrix products," *ISA Transactions*, vol. 51, no. 4, pp. 522 - 530, 2012, https://doi.org/10.1016/j.isatra.2012.03.001.

[12] Z. Yu, H. Jiang, D. Huang, and C. Hu, "Consensus of nonlinear multiagent systems with directed switching graphs: A directed spanning tree based error system approach," *Nonlinear Analysis: Hybrid Systems*, vol. 28, pp. 123-140, 2018, https://doi.org/10.1016/j.nahs.2017.12.001.

[13] S. He, W. Yu, Y. Lv, Z. Wang, and D. Zhang, "Consensus for hybrid multi-agent systems with pulse-modulated protocols," *Nonlinear Analysis: Hybrid Systems*, vol. 36, p. 100867, 2020, https://doi.org/10.1016/j.nahs.2020.100867.

[14] Z. Zhang, Y. Shi, Z. Zhang, H. Zhang, and S. Bi, "Modified order reduction method for distributed control of multi-spacecraft networks with time-varying delays," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 1, pp. 79-92, 2018, https://doi.org/10.1109/TCNS.2016.2578046.

[15] Z. Zhang, Z. Zhang, and H. Zhang, "Distributed attitude control for multispacecraft via takagi-sugeno fuzzy approach," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 54, no. 2, pp. 642-654, 2018, https://doi.org/10.1109/TAES.2017.2761199.

[16] Z. Zhang, P. Chen, H.Li, and L. Li, "Cooperative optimal attitude control of multiple rigid bodies," *2019 IEEE International Conference on Industrial Cyber Physical Systems (ICPS)*, 2019, pp. 769-774, https://doi.org/10.1109/ICPHYS.2019.8780128.

[17] H. Qin, H. Chen, Y. Sun, and L. Chen, "Distributed finite-time fault tolerant containment control for multiple ocean bottom flying node systems with error constraints," *Ocean Engineering*, vol. 189, p. 106341, 2019, https://doi.org/10.1016/j.oceaneng.2019.106341.

[18] X. Qu, X. Liang, Y. Hou, Y. Li, and R. Zhang, "Finite-time sideslip observer-based synchronized path-following control of multiple unmanned underwater vehicles," *Ocean Engineering*, vol. 217, p. 107941, 2020, https://doi.org/10.1016/j.oceaneng.2020.107941.

[19] W. Zou, C. Zhou, and Z. Xiang, "Sampled-data leader-following consensus of nonlinear multi-agent systems subject to impulsive perturbations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 78, p. 104884, 2019, https://doi.org/10.1016/j.cnsns.2019.104884.

[20] T. Dong and A. Wang, "Event-triggered consensus of nonlinear multiagent systems with unknown external disturbance," *Asian Journal of Control*, vol. 20, no. 5, pp. 1928-1937, 2018, https://doi.org/10.1002/asjc.1682.

[21] G. Zhang, J. Oin, W. X. Zheng, and Y. Kang, "Fault-tolerant coordination control for second-order multi-agent systems with partial actuator effectiveness," *Information Sciences*, vol. 423, pp. 115-127,2018, https://doi.org/10.1016/j.ins.2017.09.043.

[22] M. Hussein, J. Ghommam, A. Ghodbane, M. Saad, and V. Nerguizian,, "Distributed active fault tolerant control design against actuator faults for multiple mobile robots," *Control Theory and Technology*, vol. 17, no. 4 pp. 367-381, 2019, https://doi.org/10.1007/s11768-019-9076-7.

[23] Z. Fan, B. Fan, and W. Liu, "Distributed control of dc microgrids for optimal coordination of conventional and renewable generators," *IEEE Transactions on Smart Grid*, vol. 12, no. 6, pp. 4607-4615, 2021, https://doi.org/10.1109/TSG.2021.3094878.

[24] S. Weng, D. Yue, C. Dou, J. Shi, and C. Huang, "Distributed event triggered cooperative control for frequency and voltage stability and power sharing in isolated inverter-based microgrid," *IEEE Transactions on Cybernetics*, vol. 49, no. 4, pp. 1427-1439, 2019, https://doi.org/10.1109/TCYB.2018.2803754.

[25] F. Guo, C. Wen, J. Mao, and Y.-D. Song, "Distributed secondary voltage and frequency restoration control of droop-controlled inverter-based microgrids," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 7, pp. 4355-4364, 2015, https://doi.org/10.1109/TIE.2014.2379211.

[26] S. Manaffam, M. Talebi, A. K. Jain, and A. Behal, "Intelligent pinning based cooperative secondary control of distributed generators for microgrid in islanding operation mode," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1364-1373, 2018, https://doi.org/10.1109/TPWRS.2017.2732958.

[27] F. Guo, Q. Xu, C. Wen, L. Wang, and P. Wang, "Distributed secondary control for power allocation and voltage restoration in islanded dc microgrids," *IEEE Transactions on Sustainable Energy*, vol. 9, no. 4 , pp. 1857-1869, 2018, https://doi.org/10.1109/TSTE.2018.2816944.

[28] J. Lai, X. Lu, X. Yu, and A. Monti, "Stochastic distributed secondary control for ac microgrids via event-triggered communication," *IEEE Transactions on Smart Grid*, vol. 11, no. 4, pp. 2746-2759, 2020, https://doi.org/10.1109/TSG.2020.2966691.

[29] R. Herzallah and Y. Zhou, “Probabilistic decentralised control and message passing framework for future gnd,” *International Journal of Electrical Power & Energy Systems*, vol. 131, p. 107114, 2021, https://doi.org/10.1016/j.ijepes.2021.107114.

[30] Z. Yong and W. Wei, “Decentralised coordination control strategy of the pv generator, storage battery and hydrogen production unit in islanded ac microgrid,” *IET Renewable Power Generation*, vol. 14, no. 6, pp. 1053-1062, 2019, https://doi.org/10.1049/iet-rpg.2019.0842.

[31] A. Bani-Ahmed, M. Rashidi, and A. Nasiri, “Decentralised resilient autonomous control architecture for dynamic microgrids,” *IET Generation, Transmission & Distribution*, vol. 13, no. 11, pp. 2182-2189, 2019, https://doi.org/10.1049/iet-gtd.2018.5816.

[32] V. Calderaro, G. Conio, V. Galdi, G. Massa, and A. Piccolo, ”Optimal decentralized voltage control for distribution systems with inverter-based distributed generators,” *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 230-241, 2014, https://doi.org/10.1109/TPWRS.2013.2280276.

[33] H.-J. Moon, Y. J. Kim, J. W. Chang, and S.-I. Moon, “Decentralised active power control strategy for real-time power balance in an isolated micrognid with an energy storage system and diesel generators,” *Energies*, vol. 12, no. 3, 2019, https://doi.org/10.3390/en12030511.

[34] B. Wille-Haussmann, T. Erge, and C. Wittwer, “Decentralised optimisation of cogeneration in virtual power plants,” *Solar Energy*, vol. 84, no. 4, pp. 604-611, 2010, https://doi.org/10.1016/j.solener.2009.10.009.

[35] A. J. Veldhuis, M. Leach, and A. Yang, “The impact of increased decentralised generation on the reliability of an existing electricity network,” *Applied Energy*, vol. 215, pp. 479-502, 2018, https://doi.org/10.1016/j.apenergy.2018.02.009.

[36] J. Li and D. Zhang, “Backstepping and sliding-mode techniques applied to distributed secondary control of islanded microgrids,” *Asian Journal of Control*, vol. 20, no. 3, pp. 1288-1295, 2018, https://doi.org/10.1002/asjc.1629.