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Stabilizing of Inverted Pendulum System Using Robust Sliding Mode Control

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ABSTRACT

The Inverted Pendulum is a highly nonlinear, unstable, and fast dynamic system. These characteristics makes it a popular benchmark for building and testing novel controllers. Therefore, in this study, sliding mode controller is proposed and tested on the inverted pendulum system. According to the results of the simulation experiments with a sine signal as a reference, the proposed controller can stabilize the system well and has so fast response. Moreover, we have tuned the parameters of the proposed sliding mode controller in order to eliminate the chattering effect, the overshoot, and the steady state error.

1. Introduction

The Inverted Pendulum is a highly nonlinear [1], unstable [2], fast dynamic [3], under-actuated system [4], [5], and multivariable system [6]. This system belongs to the under-actuated mechanical system, which its control inputs are more than its degrees of freedom [7], [6], [8], [9], [10]. This characteristic encouraged many researchers to use it as a traditional benchmark for the creation, testing, assessing, and comparing of different classical and contemporary control techniques [6].

Fig. 1 shows the free body diagram of the inverted pendulum. The pendulum swings in time with the cart's movement. Even if the cart isn't moving, the pendulum stick could easily fall [7], [11]. This makes the balancing of the stick requires a fast and great force. Else, the stick may fall on the cart right away. As a result of this, the inverted pendulum controller should be designed in a way that satisfies the high response requirement. The control system's purpose is to balance the stick vertically on the cart by applying a control signal (force) on the cart [12], [13].

In literature, the inverted pendulum system is an essential element in many applications such as the balancing of robots and rocket systems when the rocket takes off [14], [15]. Consequently, researchers have suggested different linear and nonlinear controllers for balancing the inverted pendulum system. Proportional Integral Derivative Control (PID) was employed by some researchers [16], [17], [18], while Linear Quadratic Regulator (LQR) and State Feedback Control with pole placement controllers have been utilized by others [19], [20], [21], [22].

The nonlinear inverted pendulum system can be linearized using any linearization method then it can be controlled using linear controllers such as PID and state feedback. Nonetheless, the linearized

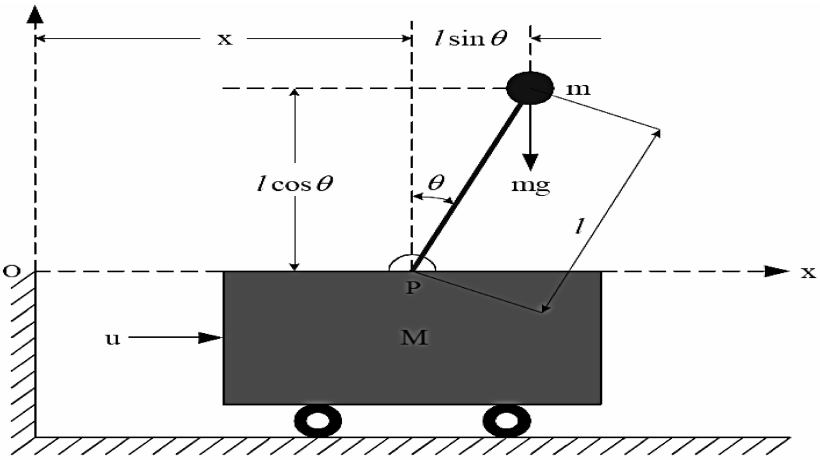

Fig. 1. Free Body Diagram

model is only a close approximation to the equilibrium with narrow angles; it cannot fully capture the system's dynamics. As a result, a linear controller can not satisfy the required response behaviour. Taking this into consideration, a nonlinear robust controller is a better choice for the inverted pendulum system.

With the recent advancement of Artificial Intelligence fields such as Fuzzy Logic Models, Artificial Neural Networks, and Evolutionary Computational Algorithms, researchers have proposed different intelligence controllers for the inverted pendulum system [23], [24]. Although the outperformance of these models upon the linear controllers, they are computationally expensive [6]. This fact motivates us to propose in this research a nonlinear robust controller which is computationally inexpensive.

Sliding mode controller (SMC) is one of the common robust controllers which has been widely utilized in the literature due to its robustness [25], [26], [27]. The basic goal of SMC is to drive the error state variables of the controlled system toward zero by utilizing a discontinuous control signal. When big control gains are utilized, the chattering effect may be activated [28]. Furthermore, the vibrations that occur can be destructive to the system [29]. Consequently, chattering effect should be taken into consideration when we design SMC [30].

The contribution of this study is to develop a suitable SMC for a nonlinear inverted pendulum system. The proposed SMC is robust to parameter uncertainty and has no chattering effect.

This article consists of four sections as follows. The first section is an introduction, which includes information about the research's background. The second section discusses both the modeling of the system and the designing of the proposed controller. The results and discussions section is the third section. The final portion contains conclusions and recommendations for future work.

2. Method

2.1. Modeling

The free body diagram of the inverted pendulum is shown in Fig. 1. The system equations of this nonlinear dynamic system can be derived as follows [7]. It is assumed here that the pendulum rod is mass-less, and the hinge is frictionless. The cart mass and the ball point mass at the upper end of the inverted pendulum are denoted as M and m, respectively. There is an externally x-directed force on the cart, $u(t)$, and a gravity force acts on the point mass at all times. The coordinate system considered is shown in Fig. 1, where $x(t)$ represents the cart position and $\theta(t)$ is the tilt angle referenced to the vertically upward direction.

The torque on the mass due to the acceleration force is balanced by the torque on the mass due to the gravity force. The resultant torque balance can be written as follows.

$$F_x lcos\theta - F_y lsin\theta = mglsin\theta \tag{1}$$

where the force components in x-axis and y-axis, F_x and F_y are determined as follows.

$$F_x = m(\ddot{x} - lsin\theta\dot{\theta}^2 + lcos\theta\ddot{\theta}) \tag{2}$$

$$F_y = -m(l\dot{\theta}^2 cos\theta + l\ddot{\theta}sin\theta) \tag{3}$$

Substituting (2) and (3) into (1) we get

$$\ddot{x}cos\theta + \ddot{\theta} = gsin\theta \tag{4}$$

A force balance in the x-direction gives that the mass times acceleration of the cart plus the mass times the x-directed acceleration of the point mass must equal the external force on the system. This can be written as follows.

$$(M+m)\ddot{x} - ml\dot{\theta}^2 sin\theta + mlcos\theta\ddot{\theta} = u \tag{5}$$

By manipulating (4) and substituting into (5) we get

$$(M+m)(gsin\theta - \ddot{\theta}) - ml\dot{\theta}^2 cos\theta sin\theta + ml\ddot{\theta}cos^2\theta = ucos\theta \tag{6}$$

which could be rewritten as follows

$$(mlcos^2\theta - (M+m))\ddot{\theta} = ucos\theta - (M+m)gsin\theta + ml\dot{\theta}^2 cos\theta sin\theta \tag{7}$$

Finally, dividing by the lead coefficients of (7) we get

$$\ddot{\theta} = \frac{-(m+M)g\sin\theta + ml\omega\cos\theta\sin\theta + u\cos\theta}{ml\cos^2\theta - (M+m)} \tag{8}$$

State variables of the system are the angular position of the oscillation and the angular velocity as represented in $x_1 = \theta$ and $x_2 = \dot{\theta}$. The input variable of the system is the applied force u, and the output of the system is the angular position as $y = x_1$. The system's disturbance and the function of parameter uncertainty term $d(x,t)$ is added as well. The full nonlinear dynamics model of the Inverted Pendulum can be written as follows.

$$\dot{x_1} = \dot{\theta} = x_2 \tag{9}$$

$$\dot{x_2} = \frac{-(m+M)g\sin x_1 + mlx_2\cos x_1\sin x_1 + u\cos x_1}{ml\cos^2 x_1 - (M+m)} + d(x,t) \tag{10}$$

2.2. Sliding-Mode Controller

Assume r_1 is the desired pendulum angle. In order to design a sliding mode controller for the Inverted Pendulum System we will follow the following procedure.

First, given that $e_1 = r_1 - x_1$, the following sliding variable is selected.

$$s_1 = \dot{e_1} + c_1 e_1 \tag{11}$$

By taking the derivative of (11) with respect to time we get.

$$\dot{s_1} = \ddot{e_1} + c_1 \dot{e_1} \tag{12}$$

By substituting the first and second derivative of the error e_1 with respect to time we can get.

$$\dot{s_1} = \ddot{r_1} - \ddot{x_1} + c_1 \dot{r_1} - c_1 \dot{x_1} \tag{13}$$

By substituting (10) into (13) we get.

$$\dot{s_1} = \ddot{r_1} - c_1 x_2 + c_1 \dot{r_1} + \frac{(m+M)g \sin x_1}{ml \cos^2 x_1 - (M+m)} - \frac{ml x_2 \cos x_1 \sin x_1 + u \cos x_1}{ml \cos^2 x_1 - (M+m)} - d(x,t) \tag{14}$$

Now, we can write the sliding mode control signal as follows.

$$u = (m+M)g \tan x_1 - ml x_2 \sin x_1 + \frac{(-\dot{s_1} + \ddot{r_1} - c_1 x_2 + c_1 \dot{r_1})(ml \cos^2 x_1 - (M+m))}{\cos x_1} \tag{15}$$

Then we can write the proposed signal control as follows.

$$u = (m+M)g \tan x_1 - ml x_2 \sin x_1 + \frac{(k_1 sign s_1 + \ddot{r_1} - c_1 x_2 + c_1 \dot{r_1})(ml \cos^2 x_1 - (M+m))}{\cos x_1} \tag{16}$$

where $sign(s_1)$ is the sign function of the sliding variable; k_1 and c_1 are any positive constants.

Now we can check the stability of the proposed controller system using Lyapunov equation. We can start defining Lyapunov equation as follows.

$$V_1 = \frac{s_1^2}{2} \tag{17}$$

The derivative of Lyapunov equation (17) is shown in (18).

$$\dot{V_1} = \dot{s_1} s_1 \tag{18}$$

From (14) and (18) we get.

$$\dot{V_1} = s_1(\ddot{r_1} - c_1 x_2 + c_1 \dot{r_1} - d(x,t) + \frac{(m+M)g \sin x_1}{ml \cos^2 x_1 - (M+m)} - \frac{ml x_2 \cos x_1 \sin x_1 + u \cos x_1}{ml \cos^2 x_1 - (M+m)}) \tag{19}$$

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By substituting the sliding mode control signal we get.

$$\dot{V}_1 = -s_1 k_1 sign(s_1) - s_1 d(x,t) \tag{20}$$

This equation can be rewritten as follows.

$$\dot{V}_1 = -k_1 |s_1| - s_1 d(x,t) \tag{21}$$

It is noticed from (21) that $\dot{V}_1 < 0$. Which means the suggested sliding mode control signal derives the system to follow the desired angular position.

3. Results and Discussion

Parameters of the inverted pendulum system are as follows. The mass of the pendulum stick is 0.1kg, the mass of the cart is 1kg, the length of the pendulum is 0.5m, and the gravity acceleration is $9.8m/s^2$. The test was made using Matlab script software. The chosen reference signal is $sin(2*pi*t)$ where $t = 0 : 0.01 : 10$.

The control parameters (c_1, k_1) for sliding mode control have been tuned using try and error. The search range of c_1 and k_1 is from 1 to 700. It is noticed from the experiments that when we increase the value of k_1 the chattering decreases. While the increase of c_1 leads to more stability. However, if c_1 is not within the range $[100 - 103]$, the system follows unstable behaviour.

Fig. 2 shows the simulation response of the experiment when $k_1 = 500$ and $c_1 = 100$. While Fig. 3 shows the state variables for the same parameters. Fig. 4 and Fig. 5 show the simulation response and state variables when $k_1 = 700$ and $c_1 = 102$. Comparing the setting of parameters in the two experiments shows that the best obtained values within the search range were when $k_1 = 700$ and $c_1 = 102$. Moreover, it is noticed from Fig. 4 that the settling time equals to $0.15sec$ and the steady state error and the maximum overshoot equal to zero.

Fig. 6 shows the simulation response of the experiment when the system is affected with disturbance equals $0.5sin(2\pi t)$, $k_1 = 700$ and $c_1 = 102$. While Fig. 7 shows the state variables for the same disturbance and parameters. Furthermore, it is noticed from Fig. 6 that the settling time equals to $0.15sec$ and the steady state error and the maximum overshoot equal to zero. Therefore, the exist of disturbance has no effect on the system which proves the robustness of the proposed controller against disturbances.

4. Conclusion

The Inverted Pendulum has been used as a popular benchmark for building and testing novel controllers in many studies. This is due to its dynamic behaviour. Therefore, in this study, sliding mode controller is proposed and tested on the inverted pendulum system. First, we have derived its mathematical dynamic model. Then we have designed a sliding mode controller to derive the angular position of the stick into the required angle. According to the results of the simulation experiments with a sine signal as a reference, the proposed controller can stabilize the system well and has so fast response. Moreover, we have tuned the parameters of the proposed sliding mode controller in order to eliminate the chattering effect, the overshoot, and the steady state error. As a future study, it is suggested to use a standard optimization method to tune the controller parameters within the suggested ranges.

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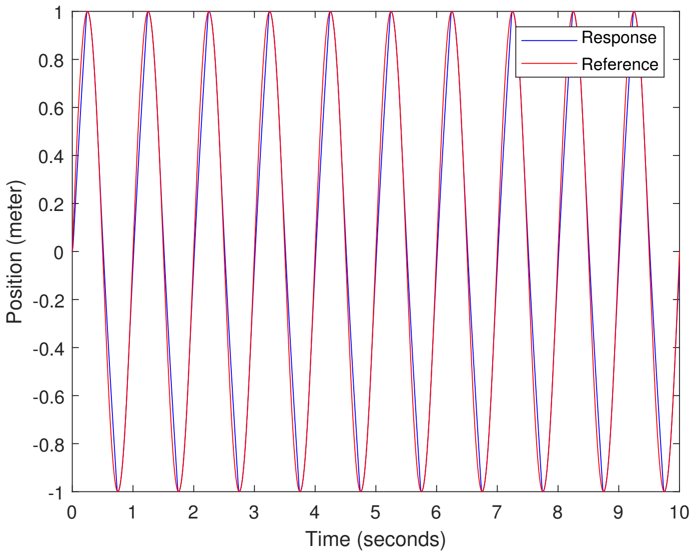

Fig. 2. System Response for Sliding Mode Control with $sin(2\pi t)$ as a Reference, $t = 0 : 0.01 : 10$, $k_1 = 500$ and $c_1 = 100$

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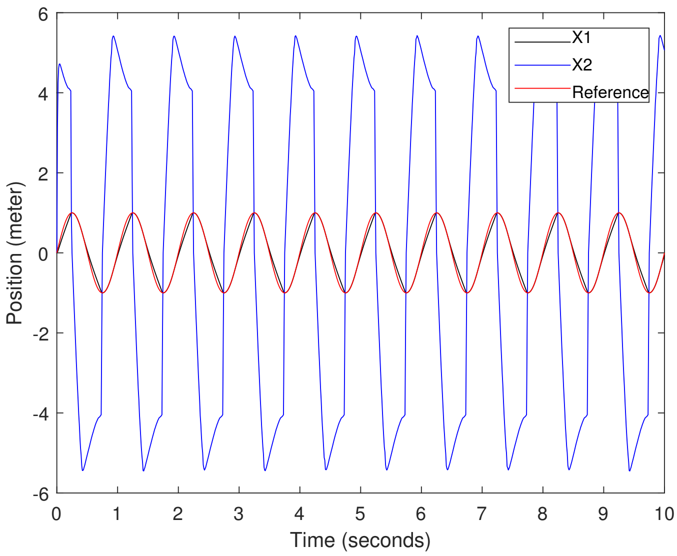

Fig. 3. State Variables for Sliding Mode Control with $sin(2\pi t)$ as a Reference, $t = 0 : 0.01 : 10$, $k_1 = 500$ and $c_1 = 100$

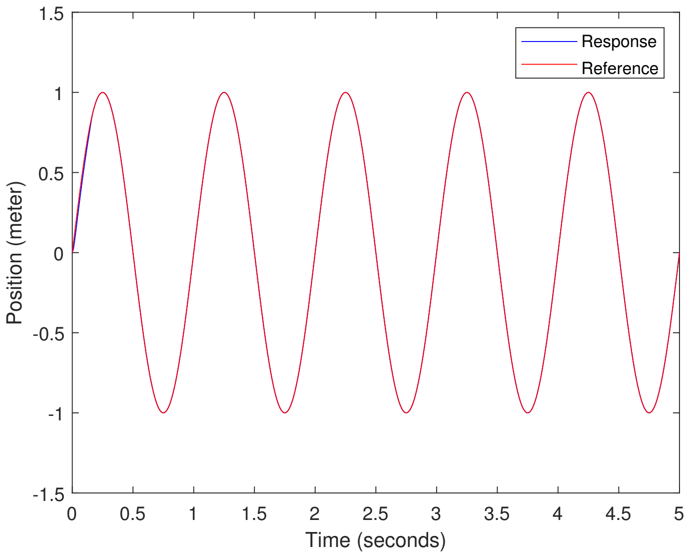

Fig. 4. System Response for Sliding Mode Control with $sin(2\pi t)$ as a Reference, $t = 0 : 0.01 : 5$, $k_1 = 700$ and $c_1 = 102$,

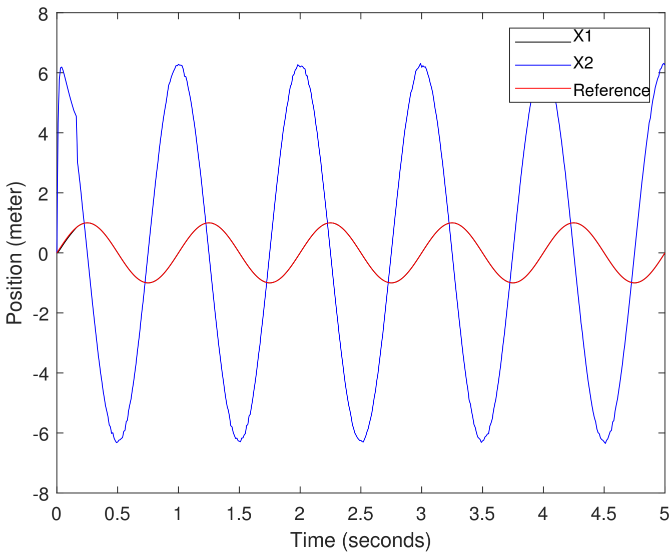

Fig. 5. State Variables for Sliding Mode Control with $sin(2\pi t)$ as a Reference, $t = 0 : 0.01 : 5$, $k_1 = 700$ and $c_1 = 102$

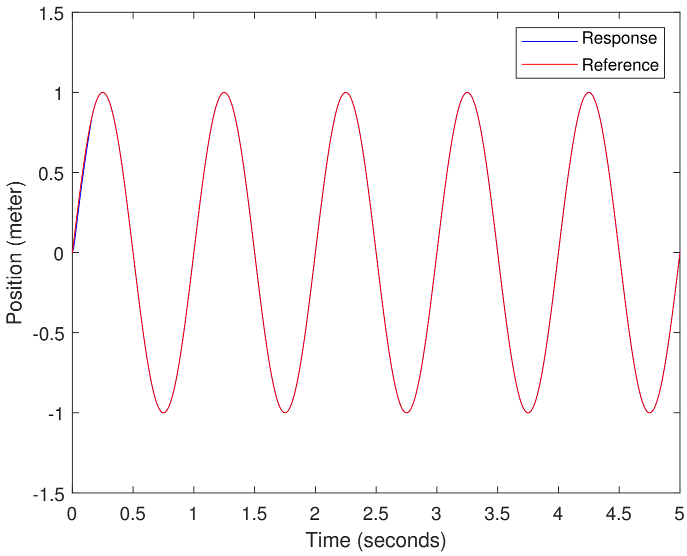

Fig. 6. State Variables for Sliding Mode Control with $sin(2\pi t)$ as a Reference, $0.5sin(2\pi t)$ as a disturbance, $t = 0 : 0.01 : 5$, $k_1 = 700$ and $c_1 = 102$

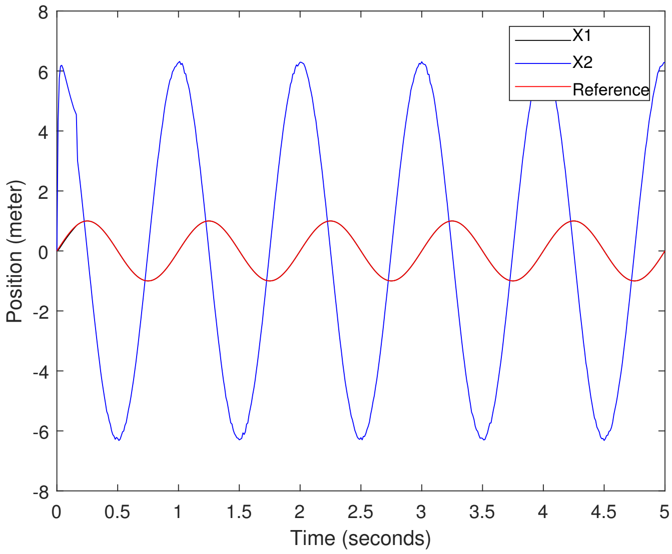

Fig. 7. State Variables for Sliding Mode Control with $sin(2\pi t)$ as a Reference, $0.5sin(2\pi t)$ as a disturbance, $t = 0 : 0.01 : 5$, $k_1 = 700$ and $c_1 = 102$

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