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# Synchronization and Chaos Control Using a Single Controller of Five Dimensional Autonomous Homopolar Disc Dynamo

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ABSTRACT

The electronic implementation, synchronization, and control of hyperchaos in a five-dimensional (5D) autonomous homopolar disc dynamo are investigated in this paper. The hyperchaotic behavior is found numerically using phase portraits and time series in 5D autonomous homopolar disc dynamo is ascertained on Orcad-PSpice software. The synchronization of the unidirectional coupled 5D hyperchaotic system is also studied by using the feedback control method. Finally, hyperchaos found in 5D autonomous homopolar disc dynamo is suppressed thanks to the designed single feedback. Numerical simulations and electronic implementation reveal the effectiveness of the single proposed control.



## 1. Introduction

Chaotic behavior has many applications in various fields of science. Unfortunately, chaotic behaviors are a source of instability and disturbance in some dynamic systems. Therefore, it is interesting to control chaotic behavior in such dynamical systems. Many authors proposed methods to suppress chaos such as resonant parametric perturbation control [1], generalized predictive control [2], adaptive control [3, 4], the input-output linearization control [5], frequency domain analysis control [6, 7], zero spectral radius control [8], optimal control [9], sliding mode control [10], single feedback control [11] and various other control methods. The single feedback control technique is concise, simple, and easy to realize. A single feedback

control method is used to control hyperchaos in 5D autonomous homopolar disc dynamo [12] in this paper.

On the other hand, since Pecora and Caroll [13] demonstrated the synchronization between chaotic coupled dynamical systems, a multitude of papers devoted to the synchronization of chaotic systems [14-18]. Several types of synchronization known as complete synchronization [13], offset synchronization [19], generalized synchronization [20-22], projective synchronization [23-27], modified projective synchronization [28], function projective synchronization [29] and various other synchronizations. A technique for synchronizing a chaotic four-dimensional system using a feedback controller, a single variable, has been proposed and demonstrated in [30]. Recently, the authors of [31] investigated the dynamics, chaos control, and synchronization in autonomous homopolar dynamo systems. The authors of [32] studied the existence of Hopf bifurcation and synchronization by using a new fuzzy controller in a 5D autonomous homopolar disc dynamo system.

Based on contributions from previous works, this paper opted to study analytically and numerically the feedback synchronization of unidirectional coupled 5D autonomous homopolar disc dynamo and chaos control using a single controller in this paper. These constitute a significant contribution to the best of our knowledge and complement of some earlier works.

The paper is structured in five sections: The rate equations and circuit design of the 5D autonomous homopolar disc dynamo are described in Section 2. A feedback synchronization of a unidirectional coupled 5D hyperchaotic system is studied in Section 3. In Section 4, the single controller is used to control hyperchaos in the 5D hyperchaotic. Section 5 concludes this paper.

## 2. Rate equations and circuit design of 5D autonomous homopolar disc dynamo

The rate equations of 5D autonomous homopolar disc dynamo are [12]

$$\frac{dx}{dt} = r\left(y - x\right) + \mathrm{w}, \tag{1a}$$

$$\frac{dy}{dt} = -\left(1+m\right)y + xz - \mathrm{v}, \tag{1b}$$

$$\frac{dz}{dt} = g\left[1 + mx^2 - \left(1+m\right)xy\right], \tag{1c}$$

$$\frac{d\mathrm{w}}{dt} = 2\left(1+m\right)\mathrm{w} + xz - k_1 x, \tag{1d}$$

$$\frac{d\mathrm{v}}{dt} = -m\mathrm{v} + k_2 y, \tag{1e}$$

Where variables $x, y, z, w, \mathrm{v}$ are the state variables and $t$ are the time, the parameters $g, m, r, k_1, k_2$ are positive reals. System (1) exhibits hyperchaotic behavior for given values of parameters, as shown in Fig. 1. The parameter is $r = 8$, $m = 0.04$, $g = 140.6$, $k_1 = 34$, and $k_2 = 12$. The initial conditions of $(x, y, z, w, v)$ are $(0.05, -0.5, 0.1, -1, 2)$. The electronic circuit of the system (1) is implemented on the Orcard-PSpice software and is depicted in Fig. 2.

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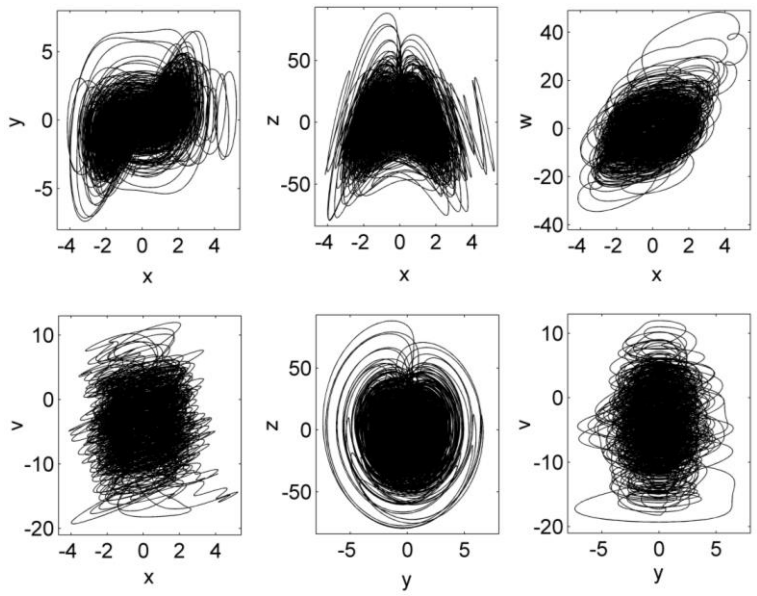


**Fig. 1.** Hyperchaotic attractor in system (1) for $r = 8$, $m = 0.04$, $g = 140.6$, $k_1 = 34$, and $k_2 = 12$.

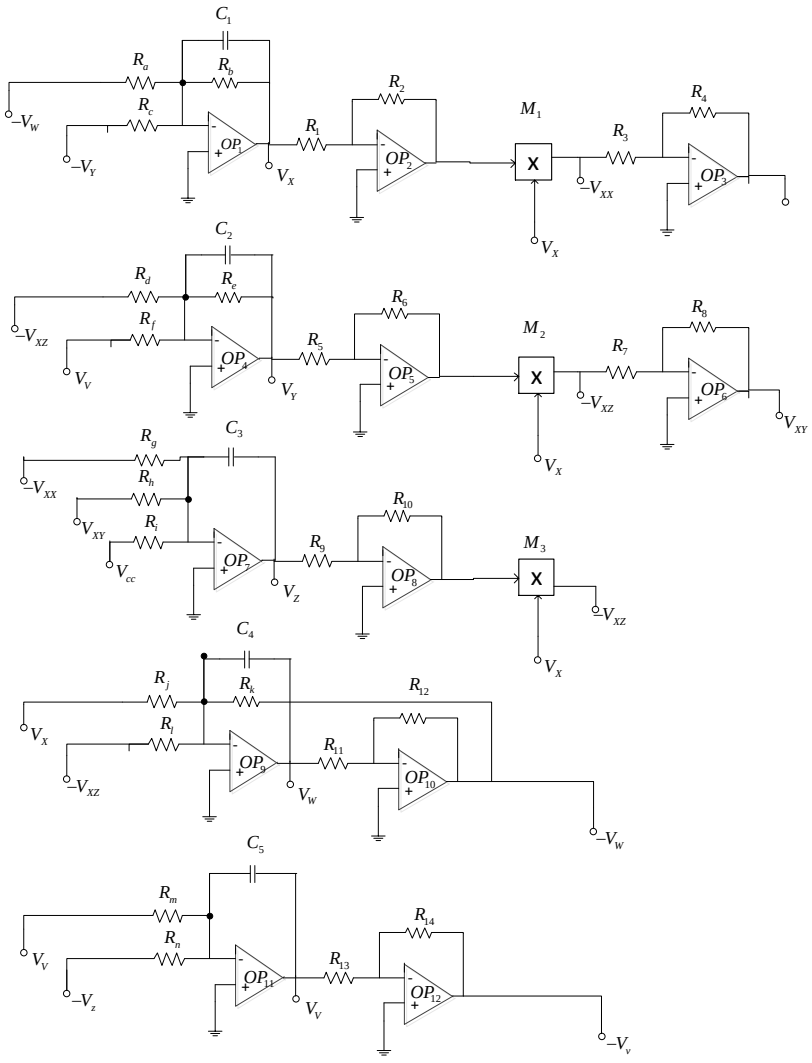


**Fig. 2.** The electronic circuit describing system (1).

The circuit of Fig. 2 is made of resistors, capacitors, operational amplifiers, and analog multiplier devices. Resistors and capacitor values are $R_a = 10k\Omega$, $R_b = 12.5k\Omega$, $R_c = 12.5k\Omega$, $R_d = 98.154k\Omega$, $C_1 = C_2 = C_3 = C_4 = C_5 = 1n\text{F}$, $R_e = 100k\Omega$, $R_f = 100k\Omega$, $R_g = 6.84k\Omega$, $R_h = 177.81k\Omega$, $R_i = 100k\Omega$, $R_j = 29.41k\Omega$, $R_k = 100k\Omega$, $R_l = 48.08k\Omega$, $R_m = 2500k\Omega$, $R_n = 83.33k\Omega$, $R_1 = R_2 = R_4 = R_5 = R_3 = 10k\Omega$, $R_6 = R_7 = R_8 = R_{10} = R_9 = 10k\Omega$, $R_{12} = R_{13} = R_{14} = R_{15} = 10k\Omega$, $V_{cc} = 14.06V$. The phase planes obtained from Fig. 2 are depicted in Fig. 3. The phase planes of Fig. 3 and the one of Fig. 1 confirm each other.

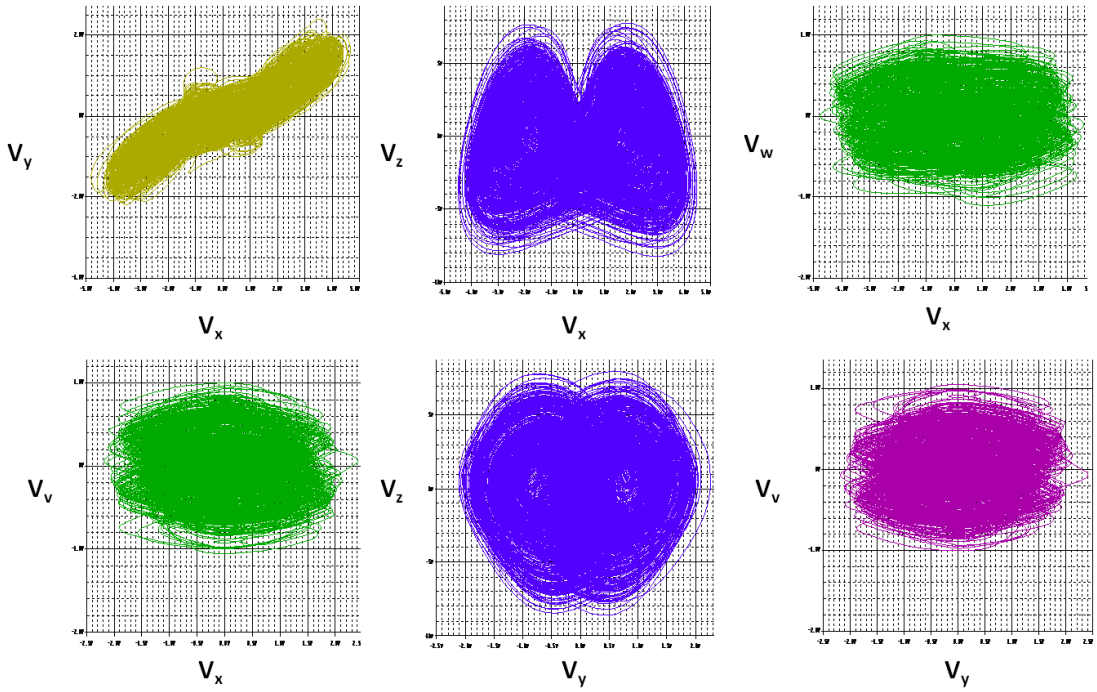


**Fig. 3.** Phase planes of hyperchaotic attractors are obtained from the electronic circuit of the system (1).

## 3. Synchronization of unidirectional coupled 5D autonomous homopolar disc dynamo

The drive and the response 5D hyperchaotic systems are expressed, respectively as

$$\frac{dx_1}{dt} = r\left(y_1 - x_1\right) + \mathrm{w}_1, \tag{2a}$$

$$\frac{dy_1}{dt} = -\left(1+m\right)y_1 + x_1 z_1 - \mathrm{v}_1, \tag{2b}$$

$$\frac{dz_1}{dt} = g\left[1 + mx_1^2 - \left(1+m\right)x_1 y_1\right], \tag{2c}$$

$$\frac{d\mathrm{w}_1}{dt} = 2\left(1+m\right)\mathrm{w}_1 + x_1 z_1 - k_1 x_1, \tag{2d}$$

$$\frac{dv_1}{dt} = -mv_1 + k_2 y_1, \tag{2e}$$

$$\frac{dx_2}{dt} = r\left(y_2 - x_2\right) + w_2 + u_1, \tag{3a}$$

$$\frac{dy_2}{dt} = -\left(1+m\right) y_2 + x_2 z_2 - v_2, \tag{3b}$$

$$\frac{dz_2}{dt} = g\left[1 + mx_2^2 - \left(1+m\right) x_2 y_2\right] + u_2, \tag{3c}$$

$$\frac{dw_2}{dt} = 2\left(1+m\right) w_2 + x_2 z_2 - k_1 x_2 + u_3, \tag{3d}$$

$$\frac{dv_2}{dt} = -mv_2 + k_2 y_2, \tag{3e}$$

Where $u_1, u_2$ and $u_3$ are the controllers. The synchronization errors are defined as follows: $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$, $e_4 = w_2 - w_1$ and $e_5 = v_2 - v_1$. Its derivatives are given as

$$\frac{de_1}{dt} = r\left(e_2 - e_1\right) + e_4 + u_1, \tag{4a}$$

$$\frac{de_2}{dt} = -\left(1+m\right) e_2 + z_2 e_1 + x_1 e_3 - e_5, \tag{4b}$$

$$\frac{de_3}{dt} = gm\left(x_1 + x_2\right) e_1 - \left(1+m\right)\left(y_2 e_1 + x_1 e_2\right) + u_2, \tag{4c}$$

$$\frac{de_4}{dt} = 2\left(1+m\right) e_4 + z_2 e_1 + x_1 e_3 - k_1 e_1 + u_3, \tag{4d}$$

$$\frac{de_5}{dt} = -me_5 + k_2 e_2. \tag{4e}$$

By choosing the controllers $u_1 = -re_2 - e_4, u_2 = -e_3 + \left(1+m\right) x_1 e_2$ and $u_3 = -3\left(1+m\right) e_4$, system (4) becomes

$$\frac{de_1}{dt} = -re_1, \tag{5a}$$

$$\frac{de_2}{dt} = -\left(1+m\right) e_2 + z_2 e_1 + x_1 e_3 - e_5, \tag{5b}$$

$$\frac{de_3}{dt} = -e_3 + gm(x_1 + x_2)e_1 - (1+m)y_2e_1, \tag{5c}$$

$$\frac{de_4}{dt} = -(1+m)e_4 + z_2e_1 + x_1e_3 - k_1e_1, \tag{5d}$$

$$\frac{de_5}{dt} = -me_5 + k_2e_2. \tag{5e}$$

The solution of (5a) is $e_1(t) = e_1(0)e^{-rt}$. So $\lim_{t\to\infty} e_1(t) = 0$ and system (5) becomes

$$\frac{de_2}{dt} = -(1+m)e_2 + x_1e_3 - e_5, \tag{6a}$$

$$\frac{de_3}{dt} = -e_3, \tag{6b}$$

$$\frac{de_4}{dt} = -(1+m)e_4 + x_1e_3, \tag{6c}$$

$$\frac{de_5}{dt} = -me_5 + k_2e_2. \tag{6d}$$

The solution of (6b) is $e_3(t) = e_3(0)e^{-t}$. Thereafter $\lim_{t\to\infty} e_3(t) = 0$ and system (6) is reduced to

$$\frac{de_2}{dt} = -(1+m)e_2 - e_5, \tag{7a}$$

$$\frac{de_4}{dt} = -(1+m)e_4, \tag{7b}$$

$$\frac{de_5}{dt} = -me_5 + k_2e_2. \tag{7c}$$

The solution of (7b) is $e_4(t) = e_4(0)e^{-(1+m)t}$. Thereafter $\lim_{t\to\infty} e_4(t) = 0$ and system (7) is reduced to:

$$\begin{pmatrix} \frac{de_2}{dt} \\ \frac{de_5}{dt} \end{pmatrix} = \begin{pmatrix} -1-m & -1 \\ k_2 & -m \end{pmatrix} \begin{pmatrix} e_2 \\ e_5 \end{pmatrix} = A \begin{pmatrix} e_2 \\ e_5 \end{pmatrix}. \tag{8}$$

For $m = 0.04$ and $k_2 = 12$, the eigenvalues of A at the equilibrium point $(e_2 = 0, e_5 = 0)$ are $\lambda_{1,2} = -0.54 \pm 3.42782730020052\,j$ with $j^2 = -1$. So, system (8) is asymptotically stable.

Therefore, the controllers $u_1 = -re_2 - e_4, u_2 = -e_3 + (1+m)x_1e_2$ and $u_3 = -3(1+m)e_4$ can synchronize the drive system (2) and the response system (3). The synchronization errors are depicted in Fig. 4. The controllers $u_1, u_2$ and $u_3$ are activated at $t \geq 1400$. The initial conditions of systems (2) and (3) are $(x_1(0), y_1(0), z_1(0), w_1(0), v_1(0)) = (0.05, -0.5, 0.1, -1, 2)$ and $(x_2(0), y_2(0), z_2(0), w_2(0), v_2(0)) = (0.5, -0.5, 0.1, -1, 2)$, respectively. Fig. 4 reveals the effectiveness of the synchronization between system (2) and system (3).

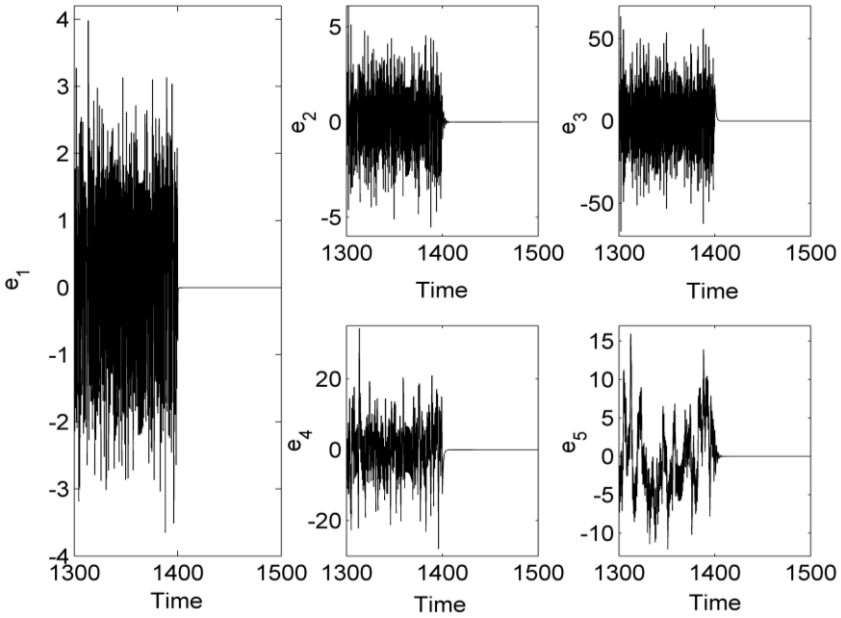


**Fig. 4.** Time series of synchronization errors for $r = 8, m = 0.04, g = 140.6, k_1 = 34$, and $k_1 = 12$.

## 4. Hyperchaoscontrol of 5D autonomous homopolar disc dynamo via a single controller

System (1) with the controller $u_4$ becomes

$$\frac{dx}{dt} = r(y - x) + \mathrm{w}, \tag{9a}$$

$$\frac{dy}{dt} = -(1+m)y + xy - \mathrm{v}, \tag{9b}$$

$$\frac{dz}{dt} = g\left[1 + mx^2 - (1+m)xy\right] + u_4, \tag{9c}$$

$$\frac{d\mathrm{w}}{dt} = 2(1+m)\mathrm{w} + xz - k_1x, \tag{9d}$$

$$\frac{d\mathrm{v}}{dt} = -m\mathrm{v} + k_2y. \tag{9e}$$

By choosing $u_4 = -z - g\left[1 + mx^2 - (1+m)xy\right]$, system (9) becomes:

$$\frac{dx}{dt} = r\left(y - x\right) + \mathrm{w}, \tag{10a}$$

$$\frac{dy}{dt} = -\left(1+m\right)y + xz - \mathrm{v}, \tag{10b}$$

$$\frac{dz}{dt} = -z, \tag{10c}$$

$$\frac{d\mathrm{w}}{dt} = 2\left(1+m\right)\mathrm{w} + xz - k_1 x, \tag{10d}$$

$$\frac{d\mathrm{v}}{dt} = -m\mathrm{v} + k_2 y. \tag{10e}$$

The solution of (10c) is $z\left(t\right) = z\left(0\right)e^{-t}$. So $\lim_{t\to\infty} z\left(t\right) = 0$ and system (10) becomes:

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{d\mathrm{w}}{dt} \\ \frac{d\mathrm{v}}{dt} \end{pmatrix} = \begin{pmatrix} -r & r & 1 & 0 \\ 0 & -\left(1+m\right) & 0 & -1 \\ k_1 & 0 & 2\left(1+m\right) & 0 \\ 0 & k_2 & 0 & -m \end{pmatrix} \begin{pmatrix} x \\ y \\ \mathrm{w} \\ \mathrm{v} \end{pmatrix} = B \begin{pmatrix} x \\ y \\ \mathrm{w} \\ \mathrm{v} \end{pmatrix}. \tag{11}$$

For $r = 8$, $m = 0.04$, $k_1 = 34$, and $k_2 = 12$, the eigenvalues of B at the equilibrium point $\left(x = 0, y = 0, \mathrm{w} = 0, \mathrm{v} = 0\right)$ are $\lambda_{5,6} = -2.96 \pm 2.9323028492978j$ and $\lambda_{5,6} = -0.54 \pm 3.42782730020052j$ with $j^2 = -1$. So, system (11) is asymptotically stable. Therefore, the hyperchaotic behavior found in the system (1) is controlled using the controller $u_4 = -z - g\left[1 + mx^2 - \left(1+m\right)xy\right]$.

The time evolutions of the state variables and the controller $u_4$ are shown in Fig. 5. The controller $u_4$ is activated at $t \geq 1300$. The initial conditions are (x(0), y(0), z(0), w(0), v(0))=(0.05,-0.5, 0.1,-1, 2). Fig. 5 demonstrates that the control of hyperchaos system (1) using the controller $u_4 = -z - g\left[1 + mx^2 - \left(1+m\right)xy\right]$ is effective. The electronic implementation of the controlled system (9) is obtained from the electronic implementation of the system (1) in Fig. 2 as shown in Fig. 6.

The circuit of Fig. 6 has 32 resistors, 5 capacitors, 13 operational amplifiers, 3 analog multiplier devices, and 1 switcher. Resistors and capacitor values are $R_a = 10k\Omega$, $R_b = 12.5k\Omega$, $R_c = 12.5k\Omega$, $R_d = 98.154k\Omega$, $C_1 = C_2 = C_3 = C_4 = C_5 = 1n\mathrm{F}$, $R_e = 100k\Omega$, $R_f = 100k\Omega$, $R_g = 6.84k\Omega$, $R_h = 177.81k\Omega$, $R_i = 100k\Omega$, $R_j = 29.41k\Omega$, $R_k = 100k\Omega$, $R_l = 48.08k\Omega$, $R_m = 2500k\Omega$, $R_n = 83.33k\Omega$, $R_1 = R_2 = R_4 = R_5 = R_3 = 10k\Omega$, $R_6 = R_7 = R_8 = R_{10} = R_9 = 10k\Omega$, $R_{12} = R_{13} = R_{14} = R_{15} = 10k\Omega$, $V_{cc} = 14.06V$.

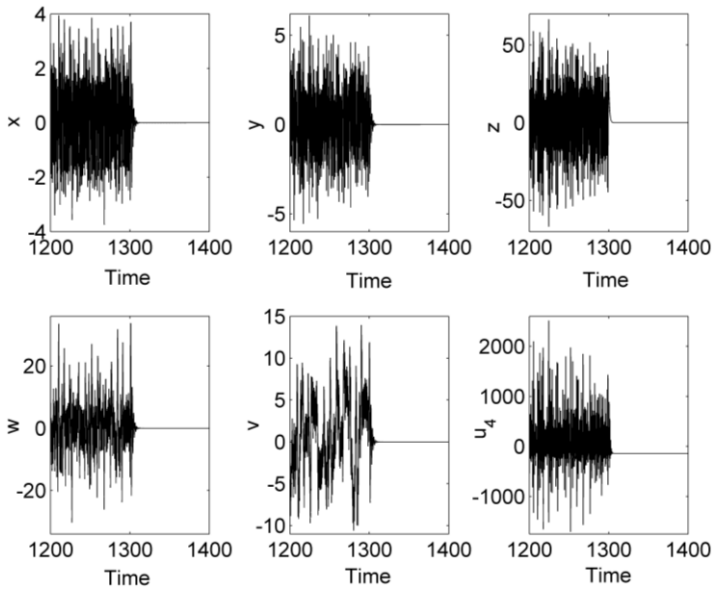


**Fig. 5.** Time evolutions of $x, y, w, v$ and $u_4$ for $r = 8, m = 0.04, g = 140.6, k_1 = 34$ and $k_2 = 12$

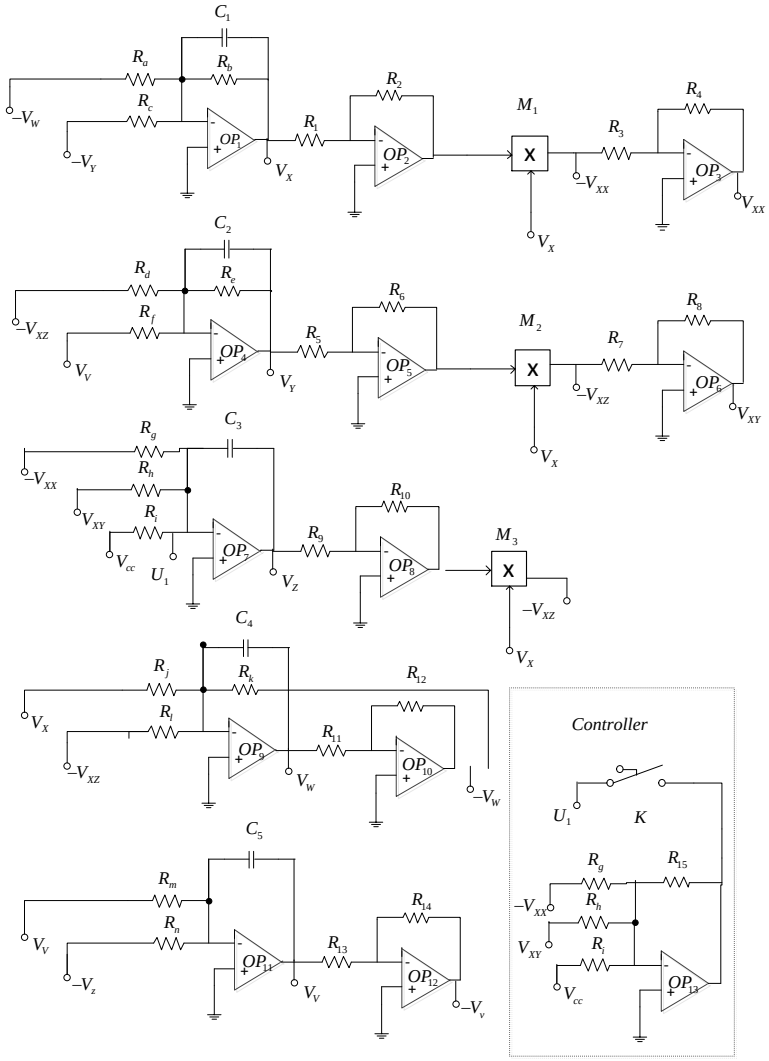


**Fig. 6.** The electronic circuit is describing the controlled system (9).

The time evolutions of the state responses and the output of the single controller generated from the circuit of the controlled system (9) are shown in Fig. 7. The time evolutions of Fig. 7 and the one of Fig. 5 confirm each other.

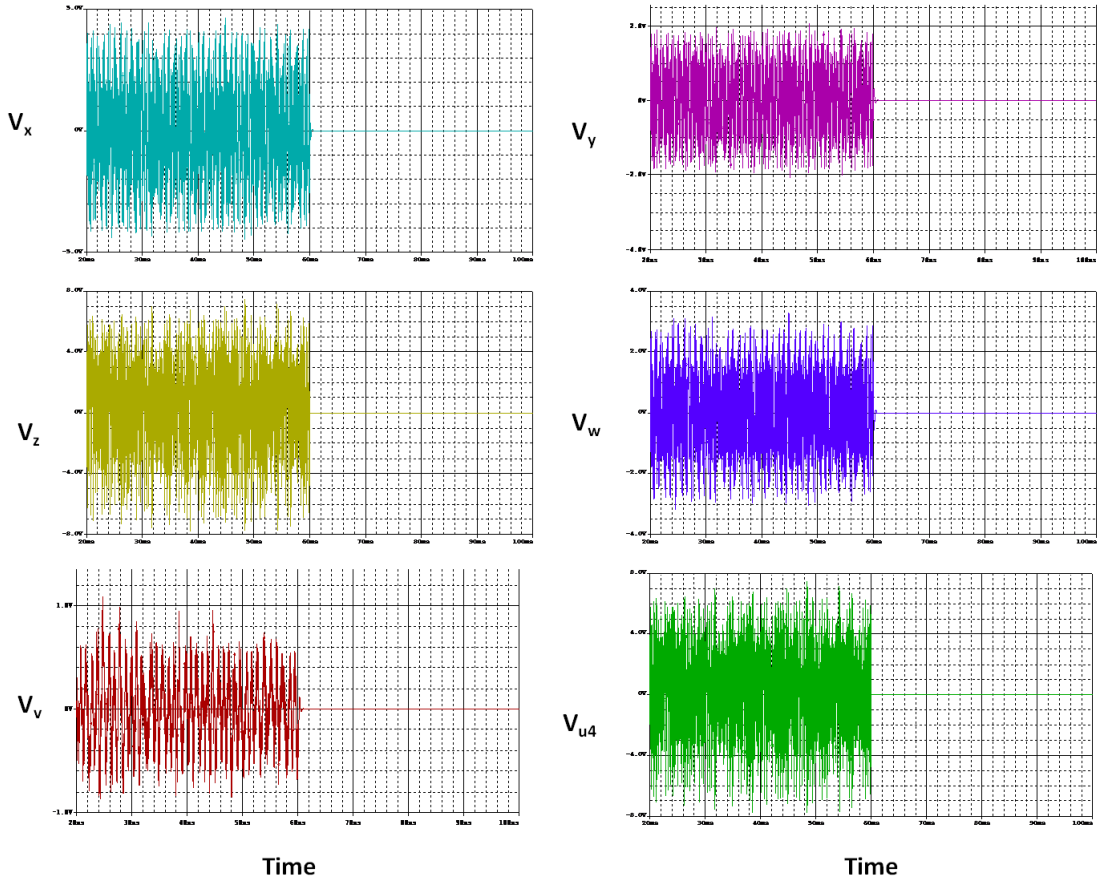


**Fig. 7.** Time evolutions of the controlled system (9) are obtained from Fig. 6.

## 5. Conclusion

The electronic implementation, synchronization, and control of hyperchaos in five-dimensional autonomous homopolar disc dynamo were investigated in this paper. The designed circuit of the five-dimensional autonomous homopolar disc dynamo was realized on Orcad-PSpice software to ascertain the hyperchaotic behavior found during the numerical simulations. The synchronization of unidirectional coupled five-dimensional autonomous homopolar disc dynamo was achieved by using the feedback control method. Finally, it was theoretically and electronically proven that the proposed single controller can control the hyperchaotic behavior of the five-dimensional autonomous homopolar disc dynamo.

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