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Adaptive Fuzzy Fault-Tolerant Control for a Class of Nonlinear Systems under Actuator Faults: Application to an Inverted Pendulum

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ABSTRACT

This work investigates a fuzzy direct adaptive fuzzy fault-tolerant Control (FFTC) for a class of perturbed single input single output (SISO) uncertain nonlinear systems. The designed controller consists of two sub-controllers. One is an adaptive unit, and the other is a robust unit, whereas the adaptive unit is devoted to getting rid of the dynamic uncertainties along with the actuator faults, while the second one is developed to handle fuzzy estimation errors and exogenous disturbances. It is proved that the proposed approach ensures a good tracking performance against faults occurring, uncertainties, and exogenous disturbances, and the stability study of the closed-loop is proved regarding the Lyapunov direct method in order to prove that all signals remain bounded. Simulation results are presented to illustrate the accuracy of the proposed technique.

1. Introduction

Recently, the Adaptive control technique has witnessed much attention in control theory society due to its ability to deal with system uncertain or unknown dynamics [1]-[9]. Generally speaking, universal approximator such as neural networks (NNs) and fuzzy logic systems (FLSs) tools was a good solution to overcome system uncertainty [10] [11], or fuzzy systems as universal approximator [1]-[6]. Various Fuzzy adaptive techniques have been developed in the literature classified from SISO to MIMO linear and nonlinear systems. In the design stage of fuzzy adaptive control law, direct and indirect approaches have been studied. In the direct, one controller consists to approximate the ideal control law with the help of a fuzzy system (see Refs. [12] [13] [14] [1]-[9]). However, the indirect resides on the approximation of the uncertain nonlinear system using fuzzy systems and based on these approximations. A general adaptive controller is built [1-5] [12] [13] [16]-[19]. On the other side, the adaptive technique was integrated with fault-tolerant control approach to handle actuator and sensor failures. Practically, sensor and/or actuator faults seem unavoidable separately or collectively due to their importance. If an actuator or sensor faults occur during the system operation, this can lead to a catastrophic behavior and drive the system to instability. Authors in [1] have investigated an adaptive fuzzy fault-tolerant control scheme for a class of nonlinear systems with simultaneous actuator and sensor failures. A combination method based on fuzzy systems (FSs) and backstepping approach allowed the online estimation of the adaptive parameters and guaranteed the boundedness of all signals in the closed-loop system, while in [4], an active fault-tolerant control technique has been proposed for a class of second-order nonlinear system subjected to state-dependent actuator faults with the presence of unknown control gain sign and external disturbances. In [20], adaptive fault-tolerant control is applied on a flexible spacecraft with state-dependent actuator failures using simple linear sets of system states and errors combination. In [21], the authors proposed a dynamic surface-based control approach using the Nussbaum-type function for attitude stabilization of a spacecraft under actuator saturation. More results can be found in [22], where an active fault-tolerant control scheme has been developed for a class of MIMO nonlinear systems with sensor failures based on dynamic surface control (DSC).

Based on the aforementioned works, a fuzzy adaptive fault-tolerant control strategy is proposed for a class on the nonlinear system with actuator faults, exogenous disturbance, and uncertainties. A modified controller with new adaptive algorithms are designed and the upper and lower bounds of the control gain sign (CGS). An additional robust control term is added to circumvent the problem of approximation errors and mollify the tracking curves.

The main contributions of the proposed controller are summarized below:

i. The proposed controller, along with a robust term, is superior to the controller performance in [14].

ii. The actuator faults model is time-varying parameters with bias, drift, loss of accuracy, and loss of effectiveness, which make the controller affordable against large faults scale.

iii. The exogenous disturbance is handled theoretically instead of approximation.

The rest of this paper is designed as follows: Problem formulation along with the studied class is first described, followed by a brief description of the universal approximation, i.e., fuzzy logic systems. Then, the proposed direct adaptive fuzzy fault-tolerant control scheme is presented with the corresponding adaptive laws and the stability analysis using Lyapunov methodology. A simulation example on the dynamic model of an inverted pendulum is performed to evaluate the accuracy of the proposed technique. Finally, some conclusions and general comments are given.

2. Problem Formulation

A class of SISO nonlinear systems without faults (faults free) can be written under the following equations [3] [6]

$$\begin{cases} \dot{x}_1 = x_2 \\ \vdots \\ \dot{x}_n = f(x_1, x_2, \dots x_n) + g(x_1, x_2, \dots x_n)u + d(t) \\ y = x_1 \end{cases} \tag{1}$$

Which can be concise and written as

$$\begin{cases} y^{(n)} = f(x) + g(x)u + d(t) \\ y = x_1 \end{cases} \tag{2}$$

where $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$, is the vector of the system; $u \in \mathbb{R}$ is the scalar control input; $y \in \mathbb{R}$ is the scalar system output; $f(x)$ and $g(x)$ are unknown smooth nonlinear functions; $d(t)$ is considered as an exogenous disturbance.

In respect to the dynamic of the system (2), the following assumptions will be made:

Assumption 1: the order n of the system is known.

Assumption 2: the state vector is available for measurement.

Assumption 3: there exists an unknown continuous positive function $D(x)$ as: $|\dot{g}(x)| \leq D(x)$

In this paper, actuator faults are considered with additive and multiplicative models, as shown in Table 1 (see in [1]).

Table 1. Actuators Faults

Actuators	Faults Kinds	Conditions	Faults Names
$\boldsymbol{u(t)}$	$u(t)+b$	$\dot{b}(t)=0, b(t_f)\neq 0$	$bias\ (Lock\ in\ place)$
	$u(t)+b(t)$	$\lvert b(t)\rvert=\lambda t\ ,0<\lambda\ll 1\ for\ all\ t\geq t_f$	$drift$
	$u(t)+b(t)$	$\lvert b(t)\rvert<\bar{b}_0\ ,\dot{b}(t)\to 0\ for\ all\ t\geq t_f$	$Loss\ of\ accuracy$
	$k(t)u(t)$	$0<\bar{k}\leq k(t)\leq 1 for\ all\ t\geq t_f$	$Loss\ of\ effectiveness$

where t_f denotes the time instant of failure of the ith sensor/actuator and b denotes its accuracy coefficient such that $b\epsilon[-\bar{b}_0,\bar{b}_0]$, where $\bar{b}_0>0$. Also $k\epsilon[\bar{k},1]$, where $\bar{k}>0$ denotes the minimum sensor and actuator effectiveness, in which b and k are slowly varying respectively within $[-\bar{b}_0,\bar{b}_0]$ and $[\bar{k},1]$.

Regarding the faults given in Table 1, then the faulty actuator can be defined by the following compact form

$$u^f(t)=k(t)u(t)+b(t) \tag{2a}$$

Meanwhile, the system described in (2a) will take the form below

$$\begin{cases} y^{(n)}=f(x)+g(x)\big(k(t)u(t)+b(t)\big)+d(t) \\ y=x_1 \end{cases} \tag{2b}$$

which can be rewritten in the following compact form

$$\begin{cases} y^{(n)}=f(x)+g(x)u(t)+f_a(x,u)+d(t) \\ y=x_1 \end{cases} \tag{2c}$$

where

$$f_a(x,u)=g(x)((k-1)u(t)+b(t))$$

The objective is to design an adaptive fuzzy controller for system (2c) under actuator faults, exogenous disturbances, and uncertainties so that the system output $y(t)$ can stably follow a referred trajectory $y_d(t)$. Its stability can be defined as all signals in the closed-loop system stay bounded.

Regarding the development of the control law, the following assumptions should also be made:

Assumption 4: the referred trajectory $y_d(t)$ and its time derivatives $y_d^{(i)}, i=1,\dots,n$ are smooth and bounded.

Assumption 5: the control gain $g(x)$ is not equal to zero, and its sign is known; $g(x)>\underline{g}>0$ with $\underline{g}$ is an unknown constant

Assumption 6: the estimation error is bounded as $|\varepsilon(x)|\leq\bar{\varepsilon}_u$

Nevertheless, the tracking error vector can be defined as

$$e = \left[e, \dot{e}, \dots; e^{(n-1)}\right]^T \in \mathbb{R}^n \tag{3}$$

with

$$e(t) = y_d(t) - y(t) \tag{4}$$

$$e^{(n)} = y_d^{(n)} - y^{(n)} \tag{5}$$

$$e^{(n)} = y_d^{(n)} - f(x) - g(x)u - f_a(x,u) - d(t) \tag{6}$$

If the functions ($f(x)$, $g(x)$, $d(t)$and $D(x)$) are known, then the control objective is achieved, and the ideal control law can be considered as:

$$u = u^* = \frac{v - f(x) - f_a(x,u) - d(t)}{g(x)} + \frac{D(x)}{2g(x)^2}\frac{e^T Pe}{e^T PB} \tag{7}$$

where P is the solution of the Lyapunov-like equation, which will be designed later, and,

$$e^{(n)} = y_d^{(n)} - f(x) - g(x)\left(\frac{v - f(x) - f_a(x,u) - d(t)}{g(x)} + \frac{D(x)}{2g(x)^2}\frac{e^T Pe}{e^T PB}\right) \tag{8}$$

$$e^{(n)} = -k^T e - g(x)\left(\frac{D(x)}{2g(x)^2}\frac{e^T Pe}{e^T PB}\right) \tag{9}$$

Hence, the dynamic error can be further written as

$$\dot{e} = Ae + B\left[-g(x)\left(\frac{D(x)}{2g(x)^2}\frac{e^T Pe}{e^T PB}\right)\right] \tag{10}$$

where

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ -k_n & -k_{n-1} & \dots & -k_n \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The Lyapunov-like equation is defined as

$$V = \frac{1}{2g(x)} e^T Pe \tag{11}$$

where P defines a symmetric positive definite matrix satisfying the Lyapunov-like equation

$$A^T P + PA = -Q \tag{12}$$

with $Q > 0$.

Finally, the time-derivative of the Lyapunov-like equation along with the error dynamic (10) can be rewritten as

$$\begin{aligned} \dot{V} = &-\frac{1}{2g(x)} e^T Qe + e^T PBg(x)^{-1}\left[-g(x)\left(\frac{D(x)}{2g(x)^2}\frac{e^T Pe}{e^T PB}\right)\right] \\ &-\frac{1}{2} e^T Pe\dot{g}(x)g(x)^{-2} \end{aligned} \tag{13}$$

Vol. 1, No. 2, 2021, pp. 102-115

This can be summarized as followed

$$\dot{V} = -\frac{1}{2g(x)}e^TQe + \dot{V}_1 \tag{14}$$

$$\dot{V}_1 = -\frac{D(x)e^TPe}{2g(x)^2} - \frac{1}{2}e^TPe\dot{g}(x)g(x)^{-2} \tag{15}$$

Based on assumption 3, the above equation can be rewritten as

$$\dot{V}_1 \leq -\frac{D(x)e^TPe}{2g(x)^2} + \left|\frac{1}{2}e^TPeg(x)^{-2}\right|D(x) = 0 \tag{16}$$

Meanwhile, based on assumption 5, equation 14 can be rewritten as

$$\dot{V} \leq -\frac{1}{2g(x)}e^TQe \leq 0 \tag{17}$$

Therefore, it can be concluded that the tracking error and its derivatives asymptotically converge to zero without any compact set $e^{(i)}(t) \to 0$ as $t \to \infty$ for $i = 0,1,\dots,n-1$ [1], and the system is globally stable. It is tough to implement the ideal control law presented in (7) since the nonlinear functions $f\,(x)$, $g\,(x)$, $D(x)$, and the exogenous disturbance are unknown. In this case, our objective is to use fuzzy systems to approach the ideal control law.

3. Fuzzy Logic Systems

It is shown and proved that any real continuous function could be approximated using fuzzy systems defined on a compact set with arbitrarily high precision [23]. Sugeno et al. [24] have proposed a class of fuzzy systems that allows representing general knowledge to be expressed in analytical form, describing the system's internal behavior.

This class of fuzzy systems is called Takagi-Sugeno (TS) fuzzy systems. The input of the fuzzy system is defined as $x = [x_1, \dots, x_n]^T$ and its output is defined as y. As for $x_i \in X_i$, each x_i is associated with m_i, fuzzy sets F_i^j in X_i. There is at least one non-zero degree of membership $\mu_{F_i^j}(x_i) \neq 0$ where $i = 1,2,\dots,n$ and $j = 1,2,\dots,m_i$.

Another notable characteristic of fuzzy systems is associated with their rules. The fuzzy system has $N = \prod_{i=1}^{n} m_i$ fuzzy rules, which have the following form

$$R_k: if\ x_1\ is\ \breve{F}_1^k\ \ and \dots and\ x_n\ is\ \breve{F}_n^k\ \ then\ \ y = f_k(x), k = 1, \dots, N \tag{18}$$

where $\breve{F}_i^k \in \{F_i^1, \dots, F_i^{m_i}\}$ and $f_k(x)$ is a numerical function on the output space. In general, $f_k(x)$ is a polynomial function depending on variable inputs, but it can also be an arbitrary function to adequately describe the studied system's behavior.

First-order and zero-order Takagi-Sugeno fuzzy systems can be distinguished from the $f_k(x)$. The $f_k(x)$ of the first-order Takagi-Sugeno fuzzy system is the first-order polynomial form, as in

$$f_k(x) = a_0^k + \sum_{i=1}^{n} a_i^k x_i \tag{19}$$

Meanwhile, the $f_k(x)$ of the zero-order Takagi-Sugeno fuzzy systems (TS-0) is a polynomial of zero-order as in

$$f_k(x) = a^k \tag{20}$$

In this work, a zero-order fuzzy system (TS-0) will be considered. A numerical conclusion is subjected to each rule. A weighted average is used to calculate the total output. In this way, the time spent on the defuzzification process can be reduced.

The output of the fuzzy system is given by the following equation [25-28]:

$$y(x) = \frac{\sum_{k=1}^{N} \mu_k(x) f_k(x)}{\sum_{k=1}^{N} \mu_k(x)} \tag{21}$$

with

$$\mu_k(x) = \prod_{i=1}^{n} \mu_{\breve{F}_i^k}$$

and

$$\breve{F}_i^k \in \{F_i^1, \dots, F_i^{m_i}\}$$

which represents the degree of confidence or activation rule R_k.

The equation (21) can be simplified and rewritten as follows:

$$y(x) = \frac{\sum_{k=1}^{N} \mu_k(x) a^k}{\sum_{k=1}^{N} \mu_k(x)} \tag{22}$$

Nevertheless, by considering the principle of fuzzy basis functions [25], the output of the TS-0 fuzzy system can be written as:

$$y(x) = w^T(x)\theta \tag{23}$$

where $\theta = [a^1 \ \dots \ a^N]$ is a vector of the fuzzy conclusion rules parameters and $w(x) = [w_1(x) \ \dots \ w_N(x)]^T$ is each component vector's basic function. The basic function $w_N(x)$ is given by:

$$w_N(x) = \frac{\mu_k(x)}{\sum_{j=1}^{N} \mu_j(x)}, k = 1, \dots, N \tag{24}$$

4. Adaptive Fuzzy Fault-Tolerant Design

This section discusses approximating the ideal control law to ensure the system can track the desired reference trajectory. A fuzzy system is used to approximate the control law with a direct approach to achieve these goals. Based on the universal approximation theory [23] of fuzzy systems, the ideal control law can be approximated as

$$u^* = w^T(x)\theta^* + \varepsilon(x) \tag{25}$$

with $\varepsilon(x)$ as the approximation error, $w(x)$ is a vector of fuzzy basis functions which was properly assumed and set in advance by the user, and θ^* is the optimal parameters' vector. The θ^* somehow minimizes $|\varepsilon(x)|$ as in

$$\theta^* = argmin_\theta\{sup_x|u^* - w^T(x)\theta|\} \tag{26}$$

The approximation error is assumed to be bounded as follows:

$$|\varepsilon(x)| \leq \bar{\varepsilon}_u$$

Vol. 1, No. 2, 2021, pp. 102-115

with $\bar{\varepsilon}_u$ is an unknown positive constant. It should be noted that the fuzzy basis functions determine the lower bound of the approximation error $\bar{\varepsilon}_u$. As a result, if these functions are better selected, the $\bar{\varepsilon}_u$ would be smaller.

Since the optimal parameters θ^* are unknown, the synthesis of the controller must be estimated. Hence, the θ is the estimation of the θ^* which will be calculated from an adaptation algorithm. The fuzzy adaptive approximation of the ideal control law is defined as

$$\hat{u} = w^T\theta + u_r \tag{27}$$

Then, the following control law can be considered based on the above equation

$$u = \hat{u} = w^T\theta + u_r \tag{28}$$

where u_r is a robust control to deal with the approximation errors. The robust control used in this work is defined as

$$u_r = sign(e^TPB)\hat{\varepsilon}_u - \frac{\sigma^2}{e^TPB} \tag{29}$$

The following are the chosen adaption laws that refer to the estimation parameters:

$$\dot{\theta} = \gamma e^T PBw(x) \tag{30}$$

$$\dot{\hat{\varepsilon}}_u = n_u|e^TPB| \tag{31}$$

$$\dot{\sigma} = -\delta_0\sigma \tag{32}$$

when σ is the time-varying parameter with $n_f > 0, \gamma > 0, \delta_0 > 0$

Remark 1 The general design of the proposed approach can be seen in Fig. 1.

Theorem:

Consider the faulty system (2c) respecting the assumptions (1-6). The control law defined by (28) and (29) with adaptions law (30-32) ensure the following properties:

- The tracking error and its derivatives converge to zero,$e^{(i)}(t) \to 0$ when $t \to\infty$ for$i = 0,1, \dots n-1$.
- The output of the system and its derivatives up to the order $(n-1)$ and the control signal are bounded: $y(t), \dot{y}(t), \dots, y^{n-1}(t), u(t) \epsilon L_\infty$.

Proof

$$\begin{aligned} e^{(n)} &= {y_d}^{(n)} - y^{(n)} \\ &= {y_d}^{(n)} - f(x) - f_a(x,u) - d(t) - g(x)u + g(x)u^* - g(x)u^* \end{aligned} \tag{33}$$

where u^* is the ideal control law, considered as an unknown term introduced just for theoretical purpose and its value is unnecessary for the design of the proposed controller.

$$e^{(n)} = {y_d}^{(n)} - f(x) - f_a(x,u) - d(t) + g(x)(u^* - u) - g(x)u^* \tag{34}$$

Replacing equation (7), equation (34) becomes

$$e^{(n)} = -k^Te + g(x)(u^* - u) - g(x)\left(\frac{D(x)}{2g(x)^2}\frac{e^TPe}{e^TPB}\right) \tag{35}$$

Meanwhile, the ideal control law is written as

$$u^* = w^T\theta^* + \varepsilon_u(x) \tag{36}$$

Let the adaptive control term considered as follows

$$u = \hat{u} = w^T\theta + u_r \tag{37}$$

Then, by combining equation (36) and (37), an equation can be found as below

$$u^* - u = w^T\tilde{\theta} + \varepsilon_u(x) - u_r \tag{38}$$

where

$$\tilde{\theta} = \theta^* - \theta \tag{39}$$

$$e^{(n)} = -k^T e + g(x)\left(w^T\tilde{\theta} + \varepsilon_u(x) - u_r\right) - g(x)\left(\frac{D(x)}{2g(x)^2}\frac{e^TPe}{e^TPB}\right) \tag{40}$$

Then, the dynamics of the error can be written as:

$$\dot{e} = Ae + B\left[g(x)\left(w^T\tilde{\theta} + \varepsilon_u(x) - u_r\right) - g(x)\left(\frac{D(x)}{2g(x)^2}\frac{e^TPe}{e^TPB}\right)\right] \tag{41}$$

where

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ -k_n & -k_{n-1} & \cdots & -k_n \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Until A is stable or $(|sI - A|) = s^{(n)} + k_1 s^{(n-1)} + \cdots + k_n$ is stable, it is known that a symmetric positive definite matrix $P\ (n, n)$ that satisfies the Lyapunov equation is existed.

$$A^TP + PA = -Q \tag{42}$$

where $Q(n\ x\ n)$ is a symmetric positive definite matrix.

The V is the Lyapunov-like equation function, then

$$V = \frac{1}{2g(x)}\ e^TPe + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2n_u}\tilde{\varepsilon}_u^2 + \frac{1}{2\delta_0}\sigma^2 \tag{43}$$

$$\begin{aligned} \dot{V} = &-\frac{1}{2g(x)}\ e^TQe \\ &+ e^TPBg(x)^{-1}\Bigg[g(x)\left(w^T\tilde{\theta} + \varepsilon_u(x) - u_r\right) \\ &- g(x)\left(\frac{D(x)}{2g(x)^2}\frac{e^TPe}{e^TPB}\right)\Bigg] - \frac{1}{2}\frac{e^TPe}{g(x)^2}\dot{g}(x) - \frac{1}{\gamma}\tilde{\theta}^T\dot{\theta} - \frac{1}{n_u}\tilde{\varepsilon}_u\dot{\hat{\varepsilon}}_u \\ &+ \frac{1}{\delta_0}\sigma\dot{\sigma} \end{aligned} \tag{44}$$

$$\dot{V} = -\frac{1}{2g(x)}\ e^TQe - \frac{1}{2}\frac{e^TPe}{g(x)^2}\dot{g}(x) + \dot{V}1 + \dot{V}2 \tag{45}$$

with

Vol. 1, No. 2, 2021, pp. 102-115

$$\dot{V}1 = e^T PBw^T \tilde{\theta} - \frac{1}{\gamma}\tilde{\theta}^T \dot{\theta} \tag{46}$$

Then

$$\dot{V}2 = e^T PB\varepsilon_u(x) - e^T PBu_r - \frac{D(x)e^T Pe}{2g(x)^2} - \frac{1}{n_u}\tilde{\varepsilon}_u\dot{\hat{\varepsilon}}_u + \frac{1}{\delta_0}\sigma\dot{\sigma} \tag{47}$$

Using equation (30)

$$\dot{V}1 = 0 \tag{48}$$

Using assumption 6

$$\dot{V}2 \leq |e^T PB|\bar{\varepsilon}_u - e^T PBu_r - \frac{D(x)e^T Pe}{2g(x)^2} - \frac{1}{n_u}\tilde{\varepsilon}_u\dot{\hat{\varepsilon}}_u + \frac{1}{\delta_0}\sigma\dot{\sigma} \tag{49}$$

Using (29), (31) and (32)

$$\dot{V}2 \leq -\frac{D(x)e^T Pe}{2g(x)^2} \tag{50}$$

The whole Lyapunov-like equation can be described as

$$\dot{V} \leq -\frac{1}{2g(x)}\, e^T Qe - \frac{1}{2}\frac{e^T Pe}{g(x)^2}\dot{g}(x) - \frac{D(x)e^T Pe}{2g(x)^2} \tag{51}$$

Based on assumption 3, the equation becomes:

$$\dot{V} \leq -\frac{1}{2g(x)}\, e^T Qe \leq -\frac{1}{2\underline{g}}\, e^T Qe \leq 0 \tag{52}$$

Hence, $V\epsilon L_\infty$ implies that the signals $e(t), \tilde{\theta}(t), \tilde{\varepsilon}(t)$ and $\delta(t)$ are bounded. This also implies that the $x(t), u(t)$, and $\dot{e}(t)$ are bounded. By using Babalat's lemma, it can be concluded that the tracking error and its derivatives converge asymptotically to zero $e^{(i)}(t) \to 0$ when $t \to \infty$ for $i = 0,1, \dots, n-1$.

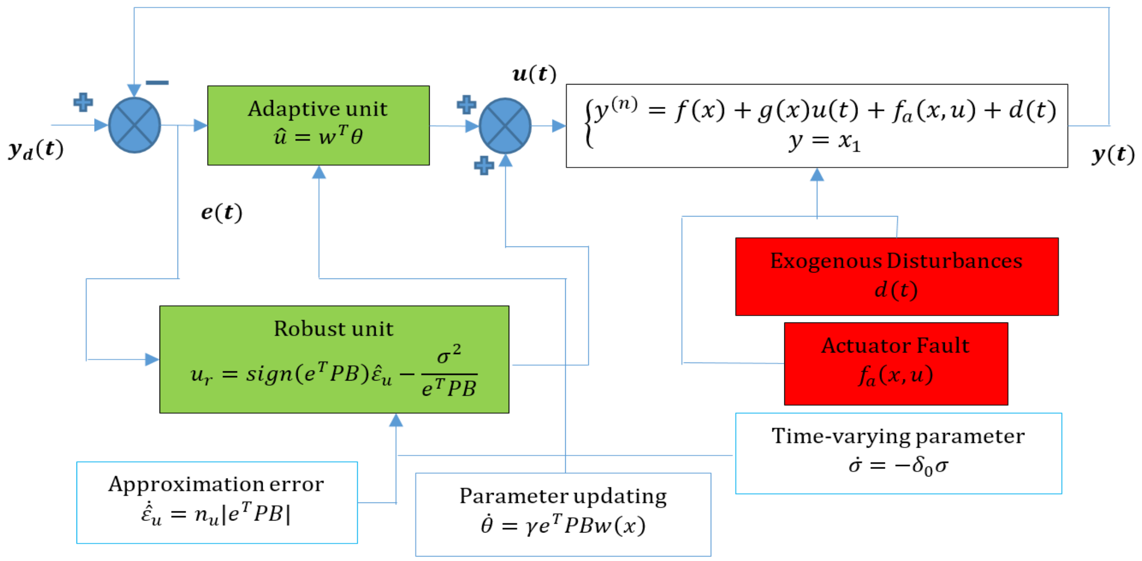

Fig. 1. The overall scheme

5. Results and Discussion

In this part, an inverted pendulum model is used to evaluate the proposed method's accuracy and prompt. The inverted pendulum mechanism system can be seen in Fig. 2. Tracking control is considered for the system. The dynamic equations of such a system are provided below as in [25].

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x)u + d(t) \\ y = x_1 \end{cases} \tag{53}$$

with

$$f(x) = \frac{gsinx_1 - (mlx_2^2cosx_1sinx_1)/(m_p + m_c)}{l(4/3 - m_pcos^2x_1/(m_p + m_c))} \tag{54}$$

$$g(x) = \frac{cosx_1/(m_p + m_c)}{l(4/3 - m_pcos^2x_1/(m_p + m_c))} \tag{55}$$

where x_1 is rotational movement, x_2 is rotational velocity, $g = 9.8m/s^2$ is the gravitational acceleration force, m_c is the cart's mass, m_p is the pole's mass, l is the pole's half-length, and u is the applied force values. Moreover, $m_c = 1kg$, $m_p = 0.1kg$ and $l = 0.5m$ are the values of the selected parameter.

The control purpose is to force the system to track the given trajectory $y_d(t) = \sin(t)$. It should be noted that the given reference allows a 1 rad maximum swing, while it is limited to 0.1 rad in [25]. Meanwhile, the exogenous disturbance is given as $d(t) = 0.45\sin(3t)$.

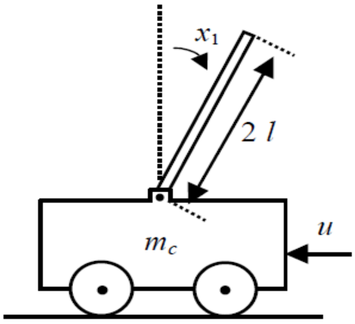

Fig. 2. The used inverted pendulum

One fuzzy system is used as an approximator for the ideal control law taking the form of (23). Two fuzzy system inputs are selected: $x_1(t)$ and $x_2(t)$. Five Gaussian membership functions are subjected to each input, and can be defined as

$$\mu_{F_i^1}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i + 1.8}{0.22}\right)^2\right)$$

$$\mu_{F_i^2}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i + 1.4}{0.22}\right)^2\right)$$

$$\mu_{F_i^3}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i}{0.22}\right)^2\right)$$

$$\mu_{F_i^4}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i - 1.4}{0.22}\right)^2\right)$$

$$\mu_{F_i^5}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i - 1.8}{0.22}\right)^2\right)$$

The initial value of the parameter $\theta(t)$ was randomly chosen. The various parameters used in this simulation were chosen as follows: $k = [1,2], P = [15\ 5; 5\ 5], B = [0; 1]\gamma = 100\ , \delta_0 = 10, n_u = 0.001$

The initial value of $\theta(0)$ was randomly selected in the range of (-2 and 2). The initial value of $\hat{\varepsilon}_u(0) = 0$ and $\sigma(0)$ =3. The simulation was conducted with actuator faults instead of sensor faults. The time profile of the faults was chosen to be at the initial time of the simulation. The actuators fault took the following forms and parameters: 1) Bias ($0.005\ N.m$); 2) Drift with $\lambda = 0.07$; 3) Loss of accuracy defined by a square waveform with $0.0087\ N.m$ amplitude and $0.15Hz$ frequency; 4) 75% loss of effectiveness.

The simulation results of the angular position $y = x_1$ and the angular velocity $\dot{y} = x_2$ are shown in Fig. 3 and Fig. 4, respectively. The control input signal $u(t)$ is shown in Fig. 5. The tracking error signal $e(t)$ is depicted in Fig. 6. We can figure out that the system output converges to the desired trajectory in a short time, even in the presence of actuator faults. So, the proposed control strategy is capable of tracking precisely.

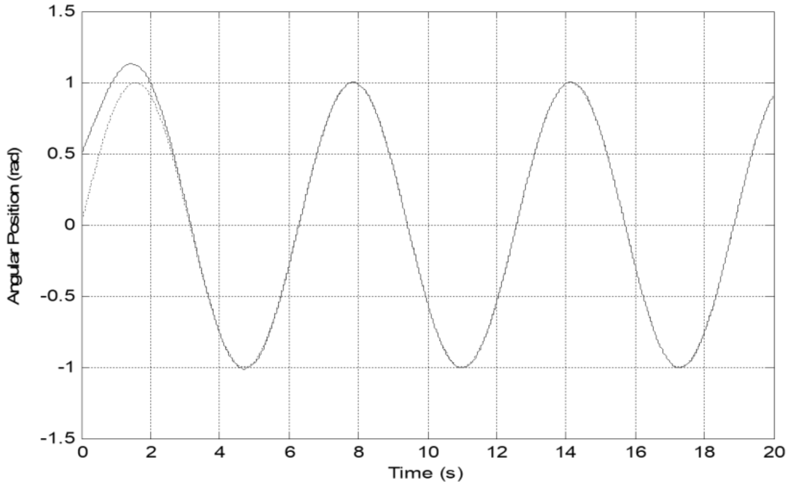

Fig. 3. $y(t)$ angular position signal (solid lines) and $y_d(t)$ reference signal (dashed lines)

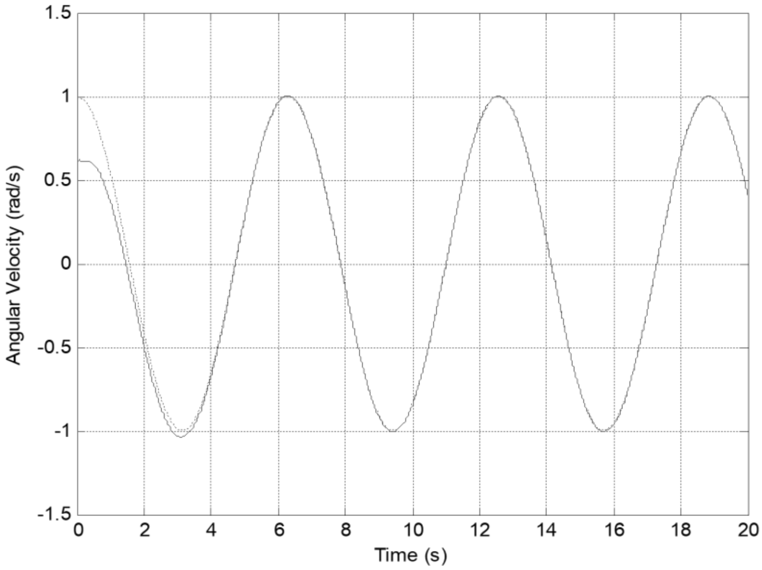

Fig. 4. $\dot{y}(t)$ angular velocity signal (solid lines) and $\dot{y}_d(t)$ reference signal (dashed lines)

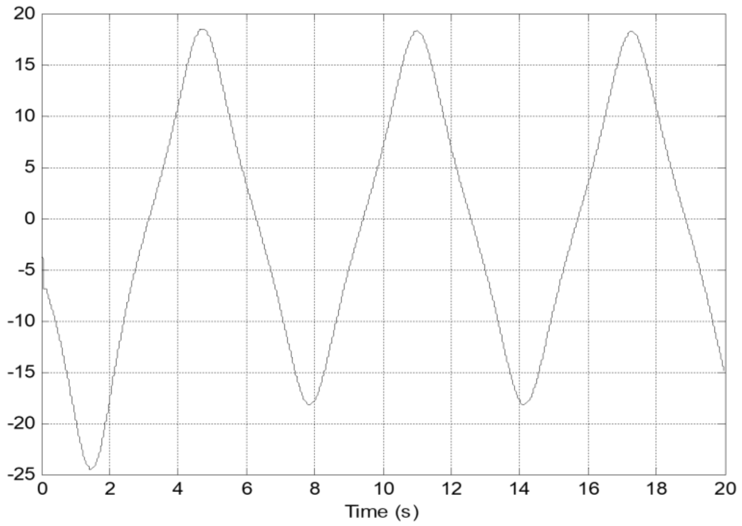

Fig. 5. Applied control input signal

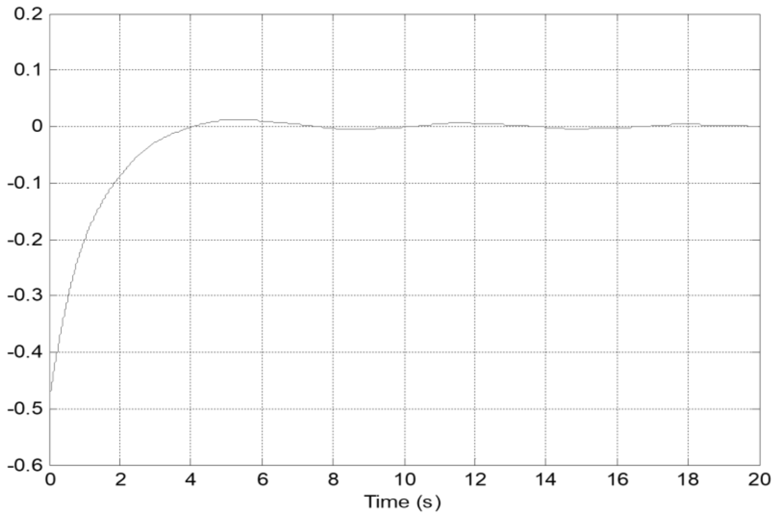

Fig. 6. Tracking error signal $(y_d(t) - y(t))$

Based on the results (Fig. 3-6), we can figure out the proposed approach reaches a good tracking performance against uncertainties, exogenous disturbances, and actuator faults. The position of the inverted pendulum $y(t)$ reaches the desired trajectory $y_d(t)$ in few seconds (around 2.5 seconds) as shown in Fig. 3 even in the presence of actuator faults, with acceptable angular velocity as depicted in Fig. 4, and the applied effort is smooth without any chattering phenomenon and acceptable power (no saturation) as shown in Fig. 5. Finally, the tracking error is closer to the origin (see Fig. 6), which implies that the control objective is reached.

6. Conclusion

The effects of time-varying actuator faults and exogenous disturbance on direct adaptive fuzzy fault-tolerant control for a class of unknown nonlinear systems are studied in this paper. Fuzzy logic systems (FLSs) are used to approach the entire adaptive control rule, including the actuator fault and the exogenous disturbance, with one robust controller term to compensate for the FLC approximation errors. The controller does not need any mathematical model of the plant, and no-fault detection and isolation FDI units are needed.

The boundedness of all signals involved in the closed-loop system and the convergence of the tracking error to zero are ensured based on the Lyapunov-like equation and Barbalat's lemma. The novelty of this paper resides in the integration of actuator faults and exogenous disturbance in the approximation of the whole adaptive controller.

Furthermore, the considered control gain is taken as a nonlinear function that extended the range of the studied systems. Moreover, our method eliminates the need for a priori knowledge of the control's gain lower bound and the approximation error's upper bounds. In the simulation section, one example applied on an inverted pendulum demonstrates the tracking performances of the proposed method.

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