International Journal of Robotics and Control Systems
Vol. 5, No. 3, 2025, pp. 1758-1769
ISSN 2775-2658
http://pubs2.ascee.org/index.php/ijrcs

Formulation of a Lyapunov-Based PID Controller for Level Control of a Coupled-Tank System

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ARTICLE INFO

Article history
Received May 20, 2025
Revised June 30, 2025
Accepted July 21, 2025

Keywords
PID Controller;
Lyapunov Stability;
Ziegler-Nichols;
Coupled-Tank

ABSTRACT

This manuscript proposes a Proportional-Integral-Derivative (PID) control algorithm based on Lyapunov stability criteria. To verify the technique, the study is further extended to investigate its feasibility in controlling the liquid level of a coupled-tank system. A comparative study is conducted with the well-established Ziegler-Nichols tuning technique, known for its rapid and aggressive response. While Ziegler-Nichols often achieves quick tuning, it is prone to instability or degraded performance, particularly in systems with slow dynamics, such as the coupled-tank system. The results demonstrate the practical viability of the Lyapunov-based PID approach. The findings show that the Lyapunov-PID controller significantly outperforms the Ziegler-Nichols PID, achieving a 33.63% reduction in overshoot and a 45.14% improvement in settling time. These improvements highlight the advantage of incorporating Lyapunov-based criteria in PID design for systems where stability and performance are critical. However, the proposed approach has limitations such as increased computational complexity and the need for abstract tuning effort, along with difficulty in selecting appropriate Lyapunov functions.

1. Introduction

A control problem can be divided into tracking and regulating cases. The regulating case forces the states to converge towards the origin upon perturbation in the initial condition [1]. Regulation can be achieved with state feedback, provided that the system is controllable. The design of feedback control systems for a tracking case is critical for ensuring the reliable operation of the system under control, such that both transient performance and steady-state performance are met. For instance, ensuring robots to track desired trajectories accurately and maintain balance [2] and tracking a solar system [3]. Among various feedback control techniques, the Proportional-Integral-Derivative (PID) controller remains widely used due to its simplicity, ease of implementation, and effective performance in a range of engineering applications. The earliest conceptual developments of three-term controllers can be traced back to 1936 [4]. This article described automatic process control systems, including concepts of feedback and early forms of three-term controllers. Later, in 1942 [5], Ziegler and Nichols formally described three-term controller as a PID control, making a significant contribution with the introduction of the Ziegler-Nichols tuning method. Nowadays, the Ziegler-Nichols tuning method remains widely used in industries such as electrohydraulic systems [6], water level control [7], [8], and advanced process control due to its simplicity and effectiveness in initializing PID parameters.

For a linear system, PID controllers are particularly advantageous for their capability to correct steady-state errors and offer satisfactory transient response. With minimal computational complexity, the stability of the closed loop control system can almost guaranteed. Advancements in PID applications can be found in adaptive tuning methods [9], position control of robotic arms [2], and adjusting the orientation of solar panel in the solar tracking system [3]. Recent advancements in PID control have been applied in various domains, including smart agriculture for precise irrigation, nutrient delivery, and climate regulation in greenhouses [10], electric vehicles for motor speed control, torque regulation, battery thermal management, and regenerative braking systems [11], HVAC systems in modern building automation [12], robotic arms in medical robotics [13], and the stable, efficient operation of microgrids [14]. Several methods are available to tune the three-term parameters of a PID controller. The conventional approaches like Ziegler–Nichols tuning method [5] and Cohen-Coon tuning remains relevant even in this decade. For instance, it has been applied in hardware-in-the-loop (HIL) control implementations for a coupled tank system using Arduino and Simulink [15]. However, these traditional tuning methods rely on system response characteristics that struggled with nonlinear or time-varying dynamics. To address these limitations, model-based self-tuning and rule-based self-tuning methods have been developed [16]. This approach improves adaptability in dynamic environments.

Though advancement of the PID design steps shows a blossom progress, the traditional method using a root locus is still relevant. Recent studies highlight the effectiveness of root locus methods in PID controller design across diverse applications. Jiang et al. [17] and Rodrigues et al. [18] emphasize educational and DC motor control benefits, while Fahmi et al. [19] extend its use to real-time human face tracking, demonstrating practical versatility. Traditional frequency-domain approaches for tuning PID parameters are still relevant today. A few decades ago, the absence of computer-aided design platforms forced designers to use hand-sketched Bode plots [20]. Recently, Barbosa, R [21] explores the application of an ideal open-loop transfer function, commonly referred to as Bode's ideal in the tuning of PID controllers. The approach uses manual straight-line Bode plots to shape the system's frequency response for desired closed-loop performance. By matching the open-loop frequency response of the system to that of a fractional-order reference model, the corresponding PID parameters can be systematically derived. This method offers significant advantages, notably enhanced robustness to plant gain variations and iso-damping characteristics, whereby the system's overshoot remains largely unaffected by changes in loop gain. Recent advancements in PID controller tuning have emphasized frequency-domain approaches for enhanced performance. Wang [22] introduced automatic tuning using frequency sampling filters, offering robust control with minimal manual intervention. Liu and Daley [23] proposed optimal frequency-domain PID tuning for rotary hydraulic systems, demonstrating improved system stability and response. Wang et al. [24] focused on performance-oriented PID tuning, integrating practical constraints to refine controller behavior. More recently, Sarman and Doğruer [25] addressed fractional-order systems, designing PID controllers with set-point filters based on time-response specifications, thus extending PID applicability to complex dynamic systems. Frequency-domain PID tuning via Bode plots targets key closed-loop criteria such as gain and phase margins, bandwidth, steady-state accuracy, overshoot control, robustness, and iso-damping behavior [26]-[29] ensuring stability and performance across variations.

Recent advancements augment artificial intelligence to enhance PID tuning such as Reinforcement learning algorithms like a Twin Delayed Deep Deterministic Policy Gradient (TD3). This tuning concept can be applied to nonlinear systems and demonstrating superior performance over classical methods [30]. To handle uncertainties and disturbances, a hybrid neural structure based on actor-critic models is employed to enable real-time self-tuning in applications like quadcopter control [31]. Additionally, optimization algorithms like the Jellyfish Search Optimization (JSO) have been utilized for tuning Fractional Order PID (FOPID) controllers [32]. The results reported JSO gives better control performance compared to traditional PID controllers. Brief literature regarding the augmentation of AI-based tuning methods concludes that this tuning approach offer adaptive, model-free solutions that enhance control performance in complex and uncertain systems, marking a significant shift in PID tuning practices.

This manuscript explores the feasibility of using Lyapunov stability criteria to tune the three parameters of a PID controller and evaluates its performance in comparison to classical techniques. Although Lyapunov's theorem is essential for assessing the stability of nonlinear controllers such as backstepping [33], [34] and sliding mode control [35], [36], it is hoped that the approach presented in this manuscript will open new avenues in control system knowledge and methodology, serving as a valuable reference for future research in this field. The foundational work introducing Lyapunov stability theory is Aleksandr Mikhailovich Lyapunov's 1892 doctoral dissertation, which was later translated into English and reprinted in 1922 [37].

Lyapunov stability theorem provides a powerful mathematical framework to guarantee the stability of dynamic systems through a Lyapunov function to ensure that the system's trajectories converge to a desired equilibrium point. The advantage of applying Lyapunov-based methods is the ability to design controllers that ensure global or local asymptotic stability without requiring exact solutions to the system dynamics [38].

There are many recent works on augmenting a Lyapunov with PID controller. For instance, author in [39] deals with the selection and evaluation of FOPID criteria for the X-15 adaptive flight control system (AFCS), using Lyapunov functions and optimization algorithms to balance trade-offs in control performance and stability. It emphasizes the theoretical and analytical aspects of designing FOPID controllers to ensure robust, stable flight control during high-speed and high-stress conditions. In [40], the author extends this idea to a practical application involving a 3-DOF helicopter model, where the effectiveness of FOPID controllers tuned using three different optimization algorithms is evaluated. The study focuses on the dynamic control of helicopter motion, aiming to enhance response accuracy and robustness under real-world disturbances. The study by author in [41] proposes a Lyapunov exponent-based PID control approach for noisy chaotic systems, demonstrating improved stability and noise resilience. This method offers a novel perspective in controller design, particularly for systems affected by chaotic behavior and external disturbances.

While augmenting Lyapunov stability in the tuning of PID controllers offers notable advantages such as improved system stability and enhanced transient performance, it is important to recognize the associated trade-offs. One of the key challenges lies in the difficulty of selecting an appropriate Lyapunov function, which is critical to ensuring the validity of the stability analysis and the overall controller performance. The process often requires deep mathematical insight and may not be straightforward for complex or nonlinear systems. Additionally, implementing a Lyapunov-based PID controller typically involves increased computational complexity, especially when real-time adaptation or optimization is involved. These factors can limit the practicality of the approach in systems with constrained processing capabilities or where simplicity and ease of tuning are prioritized.

In this manuscript, the proposed augmentation of Lyapunov stability in PID tuning is applied to a linear system, specifically a coupled-tank system. The applicability of the method is limited to systems without parametric uncertainty, exogenous disturbances, or external perturbations. Moreover, robustness against these factors is not addressed. Therefore, the main research contribution lies in the comparative study with a linear counterpart, namely the Ziegler-Nichols tuning approach.

2. Formulation of Lyapunov-Based PID Algorithm

This subsection discusses the formulation of the three-term parameters, namely the proportional gain (K_p), derivative gain (K_d), and integral gain (K_i), of a standard PID controller. A block diagram of a PID controller is depicted in Fig. 1 [42]-[44]. The error signal is denoted as $e(t)$, while the control signal is denoted as $u(t)$.

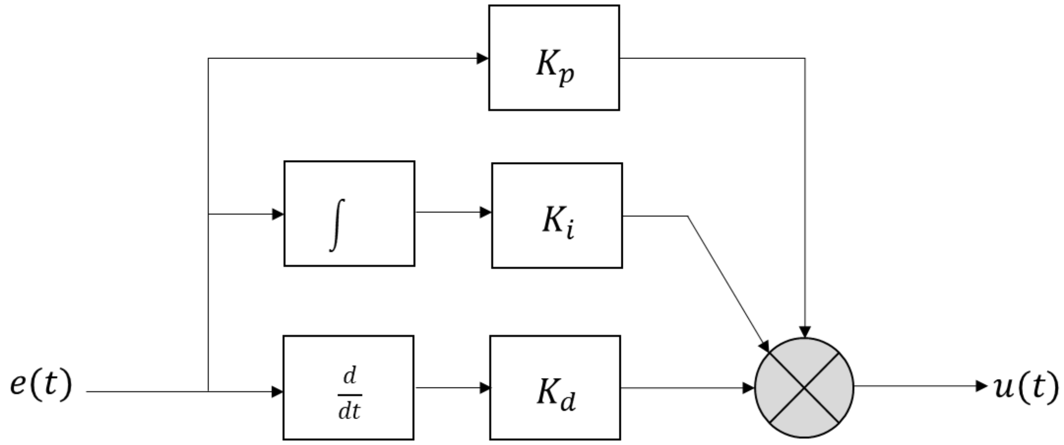


Fig. 1. PID controller

The Lyapunov stability criterion is embedded in the formulation steps to ensure the asymptotic stability of the closed-loop system. Subsequently, the efficacy of the proposed technique is verified using a numerical case study, with a DC-DC converter employed as the benchmarking model.

Consider a linear-time-invariant (LTI) system shown in equation (1), where $x(t) \in \mathbb{R}$ is the state, $u(t) \in \mathbb{R}$ is the control input. Both $f(x)$ and $g(x)$ are smooth function representing a system dynamics and input coupling function respectively.

$$\dot{x} = f(x) + g(x)u(t) \tag{1}$$

If the control input holds the dynamic of PID controller, then $u(t)$ can be expressed as in equation (2) [26]. If the $x_{ref}(t)$ is the demanded state, then the tracking error dynamic can be defined as $e(t) = x_{ref}(t) - x(t)$.

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \dot{e}(t) \tag{2}$$

The rate of change of error can be deduced as

$$\dot{e}(t) = \dot{x}_{ref}(t) - f(x) - g(x)u(t) \tag{3}$$

Assumption 1 is introduced to ease the formulation. This assumption eliminates the influence of reference dynamics, thereby allowing the analysis to focus solely on the system's response and the stability of the closed-loop error dynamics. It allows the analysis to disregard the reference signal’s rate of change (slope=0) and focus solely on the system’s output dynamics in response to a step input. Thus, facilitating stability and convergence analysis of the control system.

Assumption 1: The reference signal $\mathrm{x_{ref}(t)}$ is assumed to be a step function. This implies when $\mathrm{t} > 0$, time derivative $\dot{\mathrm{x}}_{\mathrm{ref}}(\mathrm{t}) = 0$ almost everywhere except at the discontinuity at $\mathrm{t} = 0$.

As a result, the error dynamic in equation (3) can be reduced to equation (4).

$$\dot{e}(t) = -f(x) - g(x)u(t) \tag{4}$$

For feedback stabilization, the existence of a control-Lyapunov function is necessary, as in Artstein’s theorem and Definition 1.

Artstein Theorem [45]: Artstein theorem states that a dynamical system has a differentiable control-Lyapunov function if and only if there exists a regular stabilizing feedback.

Definition 1 [24]: A smooth positive definite and radially unbounded function $\mathrm{V(x)}$ is called a Lyapunov function for a system $\dot{\mathrm{x}} = \mathrm{f(x,u)}$, such that $\frac{\partial \mathrm{V}}{\partial \mathrm{x}} \cdot \mathrm{f(x,u)} < 0$ for all $\mathrm{x} \neq 0$.

As such, a candidate Lyapunov function is chosen as in equation (5), where $\eta = \int_0^t e(\tau)\, d\tau$ is the integral of error. As such, $\dot{\eta}(t) = e(t)$, and the PID control law can be further expressed as in equation (6).

$$V(e,\eta) = \frac{1}{2}e^2 + \frac{1}{2}\eta^2 \tag{5}$$

$$u(t) = K_p e(t) + K_i \eta + K_d \dot{e}(t) \tag{6}$$

According to Lyapunov stability criteria [24], for the system in equation (1) to be stable, $V(e,\eta)$ must be positive definite ($V(e,\eta) > 0$) as in Definition 1, and its derivative must be negative semi-definite ($\dot{V}(e,\eta) \leq 0$). The derivative of a candidate Lyapunov function can be deduced as in equation (7).

$$\begin{aligned} \dot{V}(e,\eta) &= e\dot{e} + \eta\dot{\eta} \\ &= e\dot{e} + \eta e \\ &= e(\dot{e} + \eta) \end{aligned} \tag{7}$$

To ease the design concept, let assume $f(x) = 0$ and $g(x) = 1$. Hence, the error dynamics is simplified as $\dot{e}(t) = -u(t)$, and the augmentation of PID control law renders $u(t) = K_p e(t) + K_i \eta - K_d u$. Further, lumped all coefficient and embed the error dynamic into the control law, renders a control law with PID term as shown in equation (8).

$$u(t) = \frac{K_p e + K_i \eta}{1 + K_d} \tag{8}$$

Bijective mapping $u(t)$ into $e(t)$ yields

$$\dot{e}(t) = -\frac{K_p e + K_i \eta}{1 + K_d} \tag{9}$$

Equation (9) depicts the error dynamic in term of three-term PID parameters $[K_p \quad K_i \quad K_d]$. Thus, by exploiting the Lyapunov stability criteria that has been derived in equation (7), $\dot{V}(e,\eta)$ can be further expanded as

$$\begin{aligned} \dot{V}(e,\eta) &= e\left(-\frac{K_p e + K_i \eta}{1 + K_d} + \eta\right) \\ &= e\left(-\frac{K_p}{1 + K_d} e + \left(1 - \frac{K_i}{1 + K_d}\right)\eta\right) \\ &= -\frac{K_p}{1 + K_d} e^2 + \left(1 - \frac{K_i}{1 + K_d}\right) e\eta \\ &= \mathcal{M}(e) + \mathcal{N}(e,\eta) \end{aligned} \tag{10}$$

For the tracking purposes, the state $x(t) \to x_{ref}(t)$ as $t > 0$ such that the control problem can be handled like a regulation case where $e(t) \to 0$ as $t > 0$. As such, to guarantee the asymptotic stability ($e(t) \to 0$) in a sense of Lyapunov, the quadratic term $\mathcal{M}(e)$ should be negative definite, and inequality in equation (11) must be fulfilled.

$$1-\frac{K_i}{1+K_d}<0 \tag{11}$$

Then $-\frac{K_i}{1+K_d}<-1$ renders $\frac{K_i}{1+K_d}>1$. Hence, the condition of K_i and K_d can be concluded as

$$K_i>1+K_d \tag{12}$$

Then, $\mathcal{N}(e,\eta)$ becomes non-positive. To summarize, the condition for $[K_p \quad K_i \quad K_d]$ according to Lyapunov can be deduced as in Condition 1.

Condition 1: A Lyapunov-based tuning PID controller must adhere to these 3 conditions - $K_p>0$, $K_d>-1$, and $K_i>1+K_d$, such that the error e and η will converge to zero, implying asymptotic stability.

3. Analysis of Numerical Case-Level Controller

To verify the efficacy of the proposed method, a Lyapunov-based PID controller is applied to control the liquid level in a coupled-tank system. The control objective is to improve the transient performance such as overshoot and settling time, without the consideration of exogeneous perturbation and parametric uncertainties. Beforehand, a dynamic model of the coupled-tank system must be established, as the PID controller is model-based.

Akram Muntaser in [30] developed a PID controller to control the liquid level in a coupled-tank system. the coupled-tank model developed by Akram Muntaser is shown in in Fig. 2. Noted that H_1 and H_2 represent the liquid levels in tank 1 and tank 2, respectively. Q_{i1} and Q_{i2} denote the pump flow rate into tank 1 and tank 2, respectively, while Q_{o1} and Q_{o2} represent the outflow rates from tank 1 and tank 2, respectively. All the parameters are then embedded into new-defined lumped parameters to produce a transfer function in equation (13) [46].

$$G(s)=\frac{0.045}{46.159s^2+13.645s+0.65295} \tag{13}$$

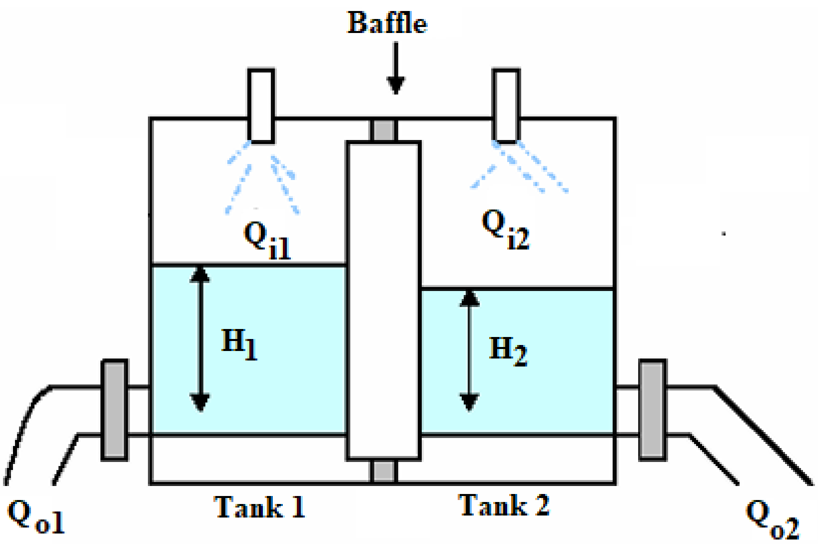


Fig. 2. Couple tank system [46]

A coupled-tank system in equation (13) has the characteristic equation of $s^2+0.2956s+0.01414=0$. This makes the system stable overdamped as its damping ratio, ζ can be calculated as $\zeta=1.243$, with a natural frequency, $\omega_n=0.1189\ rad/s$. The open-loop step response of the coupled-tank system in Equation (13) is depicted in Fig. 3. It can be observed that the response behaves as an overdamped system that requires 70 seconds to settle at 0.0689-meters. This outcome results in a significant steady-state error of 0.9311-meters. Therefore, the system requires a compensator to eliminate the steady-state error and improve the transient performance.

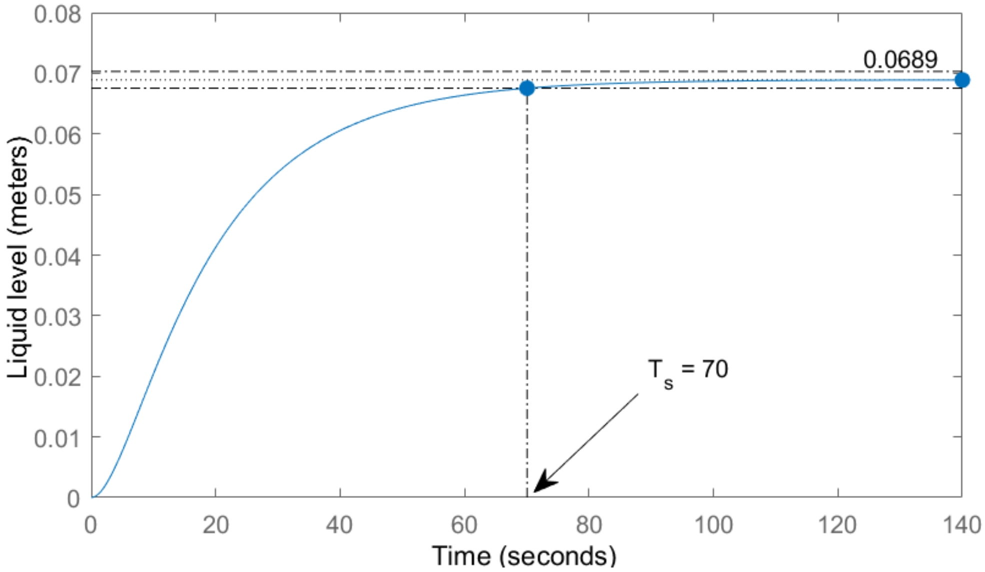


Fig. 3. Step response of open-loop coupled-tank system

For a coupled-tank system, overdamped response is acceptable as overdamped systems settle without oscillations. This is preferred since overshoot can cause spillage or unstable liquid levels. Moreover, a coupled-tank system exhibits natural damping due to the physical flow of liquid through pipes and valves, causing it to naturally behave in a sluggish, overdamped, or critically damped manner without overshooting. However, overdamping has a shortcoming as it results in a slow response by way of the liquid level rises gradually toward the desired setpoint. If the control objective is to fill the tank quickly and a slight overshoot can be tolerated, an overdamped system may be too slow and therefore requires a more responsive control strategy. In such cases, tuning toward critical damping ($\zeta \approx 1$), or even slight underdamping ($\zeta < 1$) can offer a better balance between speed and stability.

In [46], Akram Muntaser developed a PID controller using Ziegler–Nichols tuning. The research presented in this manuscript aims to develop a PID controller based on Lyapunov stability criteria and Condition 1, to enable a thorough comparative study between the two approaches. Table 1 shows the PID parameters developed by using these two approaches.

Table 1. PID parameters

PID parameters	Ziegler–Nichols tuning in [46]	Lyapunov stability criteria - Condition 1
Proportional, K_p	83.5	83.5
Integral, K_i	14.5	4.01
Derivative, K_d	120	3

To observe the feasibility of both tuning techniques, the closed-loop coupled-tank system is injected with a 0.5-meter input. The tracking history is depicted in Fig. 4. Both the Ziegler–Nichols and Lyapunov methods are able to track the desired setpoint with zero steady-state error. However, the transient performances produced by these two methods differ. It can be observed that both the Ziegler–Nichols and Lyapunov methods result in an underdamped response. However, the Lyapunov method yields a faster settling time with lower overshoot. Table 2 summarizes the results along with the percentage improvements.

Fig. 5 shows the root locus of the closed-loop coupled tank system with PID controller. By using a Lyapunov technique (Fig. 5 (a)), the locus lies entirely in the left half of the s-plane, indicating all poles remain in the stable region for a wide range of gain values (noted as K). The gain trajectory starts from a pole at the origin and extend leftward to very large negative real values, with maximum visibility at around –80. This implies that the system is stable for all gain $K > 0$.

Table 2. Transient performance of a closed-loop coupled-tank system

Performance criteria	Ziegler–Nichols [46]	Lyapunov stability criteria - Condition 1	Improvements (%)
Setting time	45.2 seconds	30 seconds	33.63%
Overshoot	35%	19.2%	45.14%

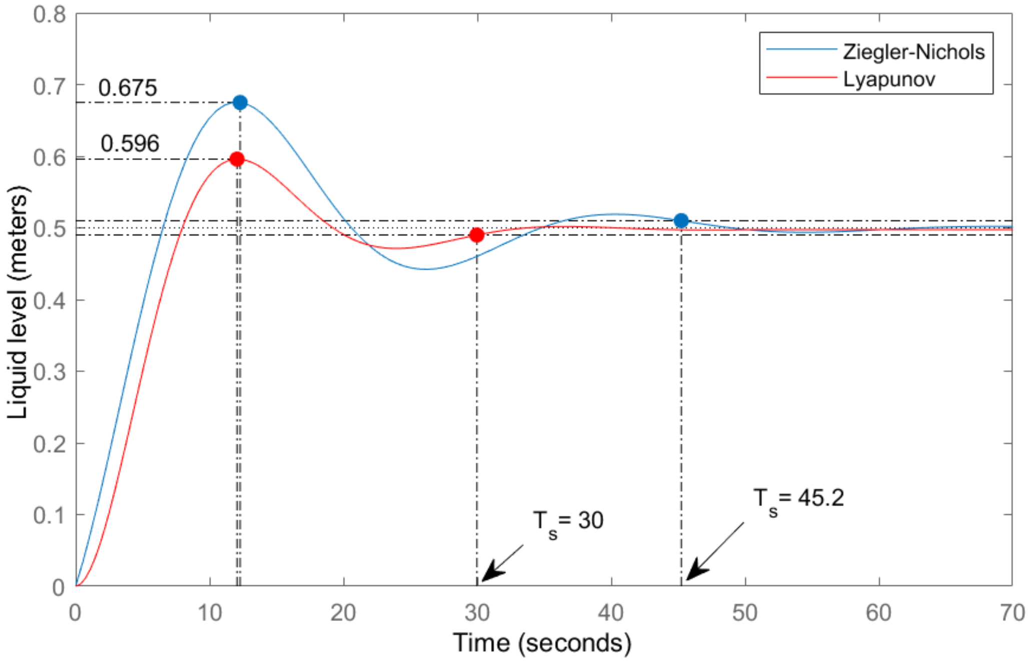
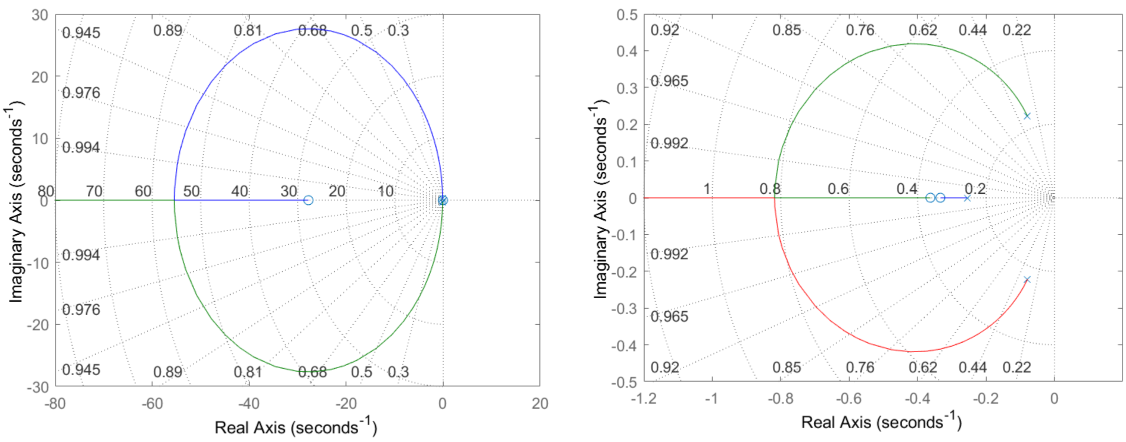


Fig. 4. Response to a 0.5-meters setpoint



(a) Lyapunov-based PID (b) Ziegler-Nichols PID

Fig. 5. Root locus of a closed-loop system with PID

It also can be observed that the locus extends far along the negative real axis, implying fast response, as more negative real parts correspond to higher damping and faster settling times. The closed-loop system with Lyapunov control exhibits a root locus with a large scale on both the imaginary axis (±30) and the real axis (up to –80), indicating that the system can tolerate high gain values while maintaining stability. This characteristic makes it suitable for systems that require high robustness and minimal oscillations, such as a coupled tank system.

In contrast, Fig. 5 (b), which uses the Ziegler-Nichols PID method shows the root locus starting from poles on the left-hand plane but quickly approaching the imaginary axis. Some branches cross into the right half-plane, indicating that the system becomes unstable at higher gain values. Therefore, with Ziegler-Nichols tuning, the stability range is limited, such that the system stable only for a narrow range of gain K. The poles also exhibit significant imaginary components, which may result in an oscillatory response. Compared to the Lyapunov-based PID, the Ziegler-Nichols method produces a closed-loop system with poles that are closer to the imaginary axis, implying a slower settling time and higher overshoot.

4. Conclusion

In the analysis of the coupled-tank system, the natural frequency $\omega_n = 0.1189$ rad/s indicates that the coupled-tank system responds slowly to input changes, controller design must consider this sluggish nature. As such, it is proven that a Lyapunov-based method offers stability, while Ziegler-Nichols may need careful tuning to avoid instability or poor performance due to the system's inherent slowness. The outcome of the controller design shows that the Lyapunov-based PID offers superior stability and conservative design. Thus, this technique is ideal for systems requiring reliability and strong performance under varying conditions. In the case of a coupled-tank dynamic, it ensures fast settling time with less overshoot as compared to its benchmarking Ziegler-Nichols PID. The improved transient response due to the movement of pole along the real axis. In contrast, Ziegler-Nichols PID provides quicker and aggressive tuning but suffers from limited stability, lower robustness, and more oscillatory underdamped responses due to pole movement near or across the imaginary axis. These overall results highlight the effectiveness of a Lyapunov-based PID in achieving faster and more stable responses for a slow system like a coupled-tank, making it a better choice for control applications requiring precision and reduced oscillations. Despite its benefits in enhancing stability and performance, the Lyapunov-based PID approach presents challenges such as selecting a suitable Lyapunov function and increased computational complexity, which may limit its practicality for complex or resource-constrained systems.

Author Contribution: All authors contributed equally to the main contributor to this paper. All authors read and approved the final paper.

Funding: This research and publication fee is funded by the Centre for Research and Innovation Management (CRIM), Universiti Teknikal Malaysia Melaka, UTeM.

Acknowledgment: This research was conducted at the Simulation Laboratory, Faculty of Electrical Technology and Engineering, Universiti Teknikal Malaysia Melaka (UTeM). The authors would like to express their sincere gratitude to the Centre for Research and Innovation Management (CRIM) for the technical support and financial assistance provided throughout this study.

Conflicts of Interest: The authors declare no conflict of interest.

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