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Interval-Valued Intuitionistic Fuzzy Cosine Similarity Measures for Real World Problem Solving

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ABSTRACT

Similarity measures (SMs) are fundamental in various applications, including identifying patterns within medical data and aiding pattern recognition (PR) by quantifying the likeness between different patterns. Moreover, they play a crucial role in real-world problems such as Multiple Criteria Decision Making (MCDM), where decision-makers assess and compare alternatives based on multiple criteria simultaneously. Moreover, Cosine similarity is a measurement that quantifies the similarity between two or more objects. This study presents a comprehensive exploration of Interval-Valued Intuitionistic Fuzzy Cosine Similarity Measures ($IVIFCSM$s) as a novel technique for assessing the degree of association between objects in real-world applications. By extending traditional cosine similarity measures (CS_M) to interval-valued intuitionistic fuzzy sets ($IVIF_S$), the proposed $IVIFCSM$s offer an effective framework for handling uncertainty, ambiguity, and imprecision in decision-making processes. The research demonstrates the practical utility of $IVIFCSM$s in addressing complex issues in PR, medical diagnosis (MD), and MCDM. In contrast to established methods like Singh's, Xu's, and Luo's measures, our approach consistently generates higher similarity values, encompassing both membership (MF) and non-membership (NMF) with interval values.

1. Introduction

Classical set theory relies on a binary assessment of element MF, posing limitations in capturing the nuanced aspects of uncertainty, vagueness, inconsistency, and imprecision inherent in human nature and real-world scenarios. In addressing these complexities, alternative set theories such as fuzzy sets (F_S), intuitionistic fuzzy sets (IF_S), and $IVIF_S$ have emerged [1]. Zadeh established the idea of F_S [2] in 1965 with the intention of capturing the subjectivity and ambiguity present in real-world settings. This is accomplished by giving objects a range of MF grades. However, this approach also presents challenges, particularly in terms of its foundation and the need for a more robust theoretical framework. Atanassov [3] further developed this idea by introducing IF_S, a generalization of F_S's. Notably, expert Zadeh [4] independently introduced a significant extension known as interval-valued fuzzy sets (IVF_S), broadening the scope of F_S theory by incorporating intervals. The concept of $IVIF_S$ was introduced by Atanassov and Gargov [5] as a generalization of IF_S and IVF_S.

In contrast to using crisp values, $IVIF_S$'s employ interval values to describe the levels of MF, NMF function, and uncertainty for each element. These sets have received widespread attention in both theoretical research and practical applications, proving more effective in handling uncertain and ambiguous information in actual environments compared to previous theories like F_S's and IF_S's. The generalized nature of $IVIF_S$'s has facilitated the more efficient resolution of complex decision-making problems, PR, and MD [6].

As a result, numerous researchers have begun working on F_S's and their extensions, as well as applying them to real-world issues [7] - [10]. The following are some relevant papers:

The experts [11] have introduced a novel distance measure for $IVIF_S$'s, which is derived from the concept of distance between interval numbers. The authors [12] created a novel integrated method for handling MCDM problems with $IVIF_S$ is proposed with the help of the multi-attribute border approximation area comparison method. Guo and Xu [13] established a new mathematical framework for knowledge measure from F_S's through $IVIF_S$'s. Based on the assessment of safety management system (Fang et al. [14]) developed an energy investment risk using house of quality based on hybrid stochastic interval-valued intuitionistic fuzzy (IVIF) decision-making approach. In [15] Li et al. investigated the IVIF approximate granular structure of a target concept and propose a new algorithm and experiments for attribute reduction from the perspective of distance.

1.1. Similarity Measures

One important technique in fuzzy mathematics is the SM. It plays a crucial role in addressing real-world problems by providing quantitative measures of similarity between data instances. They enable more effective decision-making, accurate diagnoses, and reliable PR in various domains, thereby contributing to advancements in fields such as healthcare, decision science, and artificial intelligence. Moreover, the notion of measuring similarity between two things is fundamental to MD, MCDM, and PR. Applications including topic identification, document clustering, automatic scoring, duplication detection, and text categorization are just a few of the many uses for it. Measuring the degree of similarity between two items is the primary objective of SMs. Stated otherwise, the SM is a function that determines the similarity of two items are to one another. Every SM maps to either the [0, 1] or [-1, 1] range. Absolute similarity is represented by 1, whereas minimum similarity is represented by 0 or -1 [16]. There are several different SMs available in the literature right now. The current measures of similarity fall into three groups, according to the study by Experts Gomaa and Fahmy [17]: corpus-based, knowledge-based, and string-based similarity shown in Fig. 1.

Based on existing corpora, corpus-based SMs quantify the linguistic meaning and semantic similarity of terms. Hyperspace Analogue to Language, Latent Semantic Analysis, Generalized Latent Semantic Analysis, and Explicit Semantic Analysis are a few well-known measures in this field. Additionally, knowledge-based similarity uses semantic networks like WordNet—a sizable lexical database that contains English nouns, verbs, adjectives, and adverbs—to indicate the degree of similarity between words.

Additionally, string-based SMs rely on character decompositions and text sequences to assess the similarity of two text strings are to one another. Character-based examples are Longest Common Substring, Jaro Winkler, and N-grams, whereas term-based examples include Euclidean distance, Dice coefficient, Jaccard coefficient, and Cosine similarity.

1.2. Cosine Similarity Measures

Cosine similarity measure is one of the similarity measures, which is derived from Bhattacharya's distance [18] and [19], can be expressed as the inner product of two vectors divided by the product of their lengths. Essentially, it represents the cosine of the angle formed by two F_S vector representations. A unique kind of similarity measure known as the cosine of the angle between two vectors is the CS_M. The vector representation uses the MF degree in F_S's to define the CS_M for F_S's. This approach is used to suggest certain cosine measure of similarity for IF_S's which are then used for

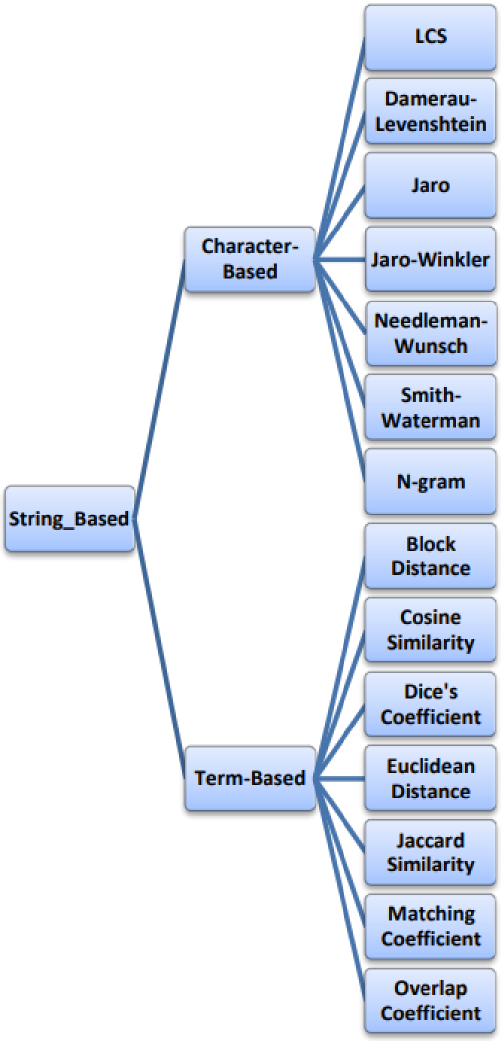

Fig. 1. String-based similarity measures

PR and MD [20]. Later, the author, Ye [21], developed a SM for the IF_S environment and created a CS_M as well as the weighted CS_M. After that, a CS_M is used by Singh and Ye [22], [23] to evaluate the similarity between two $IVIF_S$'s. Moreover, a strategy for solving group decision-making tasks was devised by Liu et al. [24], grounded on the $IVIF_S$-based weighted CS_M. In recent years, the experts [25] - [30] have developed and enhanced a CS_M for $IVIF_S$'s and applied it effectively to tackle decision-making challenges. Evaluating two distinct objects from multiple perspectives is essential when addressing a variety of real-world challenges, spanning fields like PR, decision making, image processing, and machine learning. The choice of a compatibility or comparison measure can vary depending on the specific problem domain. Among the notable compatibility and comparison measures are SMs, distance measures, and correlation measures. Extensive literature exists regarding the applications of these measures in both fuzzy and non-standard fuzzy scenarios [31], [32].

1.3. Literature Review

This paper primarily focuses on exploring SMs within the context of $IVIF_S$ The authors [34] presented innovative SMs that involve combining the exponential function of MFs with the negative function of NMFs. Additionally, within this paper, a novel entropy measure was presented as a fundamental component for determining the criteria weights in the proposed MCDM model. Further, Mishra et al. announced a low-carbon tourism strategy evaluation with selection based on SMs [35]. In addition, Lee et al. [36] built a novel MCDM approach grounded on IF_S, weighted SMs, and the extended technique for order preference by similarity to ideal solution method. Furthermore, a novel similarity among the IF_S based on the transformation techniques with their characteristics and applications are explored by Garg and Rani [37].

This article [38] introduces mathematical models of tumors used by the method of model predictive control and determines the right dosage of the drug to reduce tumor density. The authors describe a dice similarity measure for IF_S's that is implemented to various classification problems in PR and MD [39]. A measure of similarity serves as a valuable tool for recognizing patterns in various fields where the optimal decisions or criteria for optimality have been specified. Likewise, Donyatalab et al. recommended a novel SM for spherical F_S's as well as applied it to the field of MD problems [40].

Azita Mousavi et al. [41], [42] investigated the face recognition percentage of the identity check system applied by the SVM pattern recognition algorithm on the face image, and they executed this method in the ORL database. In this work, an interval agreement approach method for calculating the degree of similarity between two aggregated fuzzy numbers is presented [43]. The main findings of this article are the proposal of new similarity measures for spherical fuzzy sets and T-spherical fuzzy sets, which are applied to the PR problem [44]. The authors proposed the similarity and weighted similarity measures by considering MF, NMF, and the indeterminacy of MF degree between the q-ROFSs [45]. In this work, the authors provide new similarity measures for PFSs, which are used in PR [46].

The following are the main reasons that motivated us to consider the present study: Taking a closer look at the current SMs for $IVIF_S$'s, it becomes evident that devising a robust $IVIF_S$ SM is quite difficult. Some of these measures fail to fully adhere to the axiomatic definition of similarity, leading to counterintuitive outcomes, as demonstrated by researchers who have found counterintuitive examples for many existing measures.

There are several shortcomings in many of the current SMs for $IVIF_S$'s; the majority of them provide findings that are counterintuitive in certain circumstances and are unable to provide accurate classification results for issues related to PR and MD. To compute the SMs between P and Q_i (i = 1, 2), for example, let P = $< [0.20, 0.30], [0.40, 0.60] >$, Q_1 = $< [0.30, 0.40], [0.40, 0.60] >$, and Q_2 = $< [0.30, 0.40], [0.30, 0.50] >$ be $IVIF_S$'s. These may be found using Formulas (2), (3) and (5) see Section 3. Since the MF degree of Q_1 and Q_2 are the same, and the NMF degree of Q_1 and Q_2 differs, it follows that the outcome is $Q_1 \neq Q_2$. As a result, $S_i(P, Q_1) = S_i(P, Q_2)$ $(i = 1, 2)$. But using Formulas (2) and (3), we can get that $S_1(P, Q_1) = S_2(P, Q_1) = S_1(P, Q_2) = S_2(P, Q_2) = 0.9$, which is unreasonable. In the meantime, Formula (5) makes it possible to obtain $S_D(P, Q_1) = 1$, which does not meet the second postulate of the definition for SM. To address these shortcomings, we need to create a new similarity measurement.

Additionally, other SMs lack a defined physical interpretation and involve highly complex expressions. Initially, many of these measures were primarily focused on their mathematical formulas. Consequently, there is a need for a more comprehensible $IVIF_S$ SM. It is also worth noting that previous studies [47], [23], [22] did not incorporate the middle and boundary points of the intervals into their approaches. Therefore, formulating an effective SM remains an unresolved challenge that is attracting increased attention. Because of these limitations, the measurement procedure is unable to account for the preferences of the decision-maker. This highlights the limitations and inflexibility of these measures when applied to real-world problems. Therefore, within the context of $IVIF_S$, there is a need for a robust CS_M that can incorporate the decision maker's attitude preferences into the measurement process.

1.4. Research Objectives of this Study

The research objective of this study is to introduce and validate a novel technique for assessing the degree of association between two objects. Specifically, the focus is on developing IVIFCSMs based on the concept of cosine similarity. The aim is to create a measure that not only adheres to important practical criteria but also demonstrates its effectiveness in addressing real-world problems. The study applies this novel measure to MCDM problems, MD scenarios, and PR scenarios involving IVIF information. The research provides numerical examples showing the application of the proposed method. In brief, the study objectives are to propose a new SM for $IVIF_S$'s, present its properties, compare it with existing measures, and apply it in PR, MD and MCDM.

1.4.1. Goal and Contribution of the Study

The goal of this study is to provide an effective SM for the $IVIF_S$ environment that performs better than methods found in [22], [47] - [50]. In particular, this paper:

- Reviews some of the existing similarity techniques;
- Suggests an enhanced $IVIF_S$ similarity technique;
- Compares the new similarity technique in the $IVIF_S$ domain;
- Uses the new similarity technique to identify some real-world scenarios.

In light of the preceding information, the contribution of our approach unfolds as follows: we first develop a unique CS_M for $IVIF_S$'s based on the aspect of CS_M between $IVIF_S$'s. Second, we used the concept of a SM to validate the fundamental and necessary aspects of a suggested similarity measure. Furthermore, the suggested approach's application to several real-world issues involving IVIF information is investigated. The suggested measure's numerical examples are then compared to other sophisticated measurements to demonstrate its effectiveness.

1.4.2. Structure of the Article

The remainder of the paper is designed as follows. Section 2 summarizes the fundamental notions of $IVIF_S$ and SMs. Section 3 examines various existing similarity measures. In Section 4, we present the IVIFCSM between two $IVIF_S$'s. In addition, various features of the IVIFCSMs are investigated. Section 5 applies the IVIFCSM to real-world issues such as MD, PR, and MCDM with IVIF information, as well as demonstrative with numerical examples. Section 6 outlines the key findings and conclusions of the article.

2. Preliminaries

This section discusses the fundamental ideas associated with $IVIF_S$'s that were utilized in our work.

Definition 2.1 (F_S). [2]

A fuzzy set A in X is characterized by a MF function $\mu_A(x)$ which associates with each point in X a real number in the interval $[0, 1]$, with the value of $\mu_A(x)$ at x representing the grade of MF function of x in A.

- if $\mu_A(x) = 0$, then it represents to NMF function.
- if $0 < \mu_A(x) < 1$, then it represents to partial MF function.

- if $\mu_A(x) = 1$, then it represents the full-membership function.

A F_S can be classified into continuous F_S or discrete F_S. The continuous F_S A can be written in the form

$$A = \{\langle x, \mu_A(x)\rangle | x \in X, \mu_A(x) \in [0,1]\}.$$

The discrete F_S A can be represented in the following form

$$A = \left\langle \frac{\mu_A(x_1)}{x_1}, \frac{\mu_A(x_2)}{x_2}, ... \frac{\mu_A(x_n)}{x_n} \right\rangle, \quad \forall x_i \in X, i = 1, 2, ..., n.$$

Let us mainfest F_S's over crisp sets through the following example:
Example:

"Sathyamangalam is located in Coimbatore?". For this question, the answer is either yes (1) or no (0). Here, the MF function possibilities are zero-membership function or full-membership function only. Hence it is crisp set.

"Anu is a honest person?". For this the answer maybe any of the following, extremely dishonest (0) or extremly honest (1) or honest at time (0.3) or very honest (0.7). Hence this be relevant to the F_S.

Definition 2.2 (IF_S)**.** [3]

An IF_S of A in E is defined by the formula

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}.$$

For any $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, and the functions $\mu_A, \nu_A : E \longrightarrow [0,1]$ defines the degree of MF & NMF, respectively. The third parameter within the IF_S A is the hesitancy degree of whether x is in A or not, represented as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. This parameter is bounded between 0 to 1. Here, E is a fixed, non-F_S. The hesitation degree is a part of MF function of degree or part of NMF function of degree or both. The following example describe the above three degrees:
Example:

Let us consider the set of people in age group 17 year and above is a domain of discourse. The MF function of degree, hesitation degree and NMF function of degree of the IF_S is described as follows:

If we denote the young people in age group between $17 - 40$ year by MF function of degrees, then we denote the old people in age group 55 year and above by NMF function of degree. The people in the age group between $40 - 55$ years may be considered as young or old or both. Thus, we can represent these people (age group between $40 - 55$ year) by hesitation degree.

Definition 2.3 ($IVIF_S$)**.** [5]

Let R be the $IVIF_S$ in the universe of discourse X, then

$$R = \{\langle x, \mu_R(x), \nu_R(x)\rangle | x \in X\}.$$
$$= \{\langle x, [\mu_R^-(x), \mu_R^+(x)], [\nu_R^-(x), \nu_R^+(x)]\rangle | x \in X\}$$

In this case, $\mu_R(x), \nu_R(x) \subseteq [0,1]$ satisfy $0 \leq \mu_R(x) + \nu_R(x) \leq 1$. Moreover, the intervals $\mu_R(x)$ and $\nu_R(x)$ represent the degrees of MF & NMF, respectively. Additionally, we will calculate the degree of hesitancy for every $x \in X$ as follows: $\pi_R(x) = [\pi_R^-(x), \pi_R^+(x)] = [1 - \mu_R^+(x) - \nu_R^+(x), 1 - \mu_R^-(x) - \nu_R^-(x)]$.

Definition 2.4. [47]

Let R and T be the two $IVIF_S$'s in the universe of discourse X, then

1. $R \subseteq T$ iff $(\forall\, x \in X)\; \mu_R^-(x) \leq \mu_T^-(x), \mu_R^+(x) \leq \mu_T^+(x), \nu_R^-(x) \leq \nu_T^-(x)$, and $\nu_R^+(x) \leq \nu_T^+(x)$.
2. $R = T$ iff $(\forall\, x \in X)\; \mu_R^-(x) = \mu_T^-(x), \mu_R^+(x) = \mu_T^+(x), \nu_R^-(x) = \nu_T^-(x)$, and $\nu_R^+(x) = \nu_T^+(x)$.
3. $R^c = \{x, [\nu_R^-(x), \nu_R^+(x)], [\mu_R^-(x), \mu_R^+(x)]\}$.

Definition 2.5 (Similarity Measures)**.** [47]

Suppose $X = \{x_1, x_2, \cdots, x_n\}$ is the universe of discourse. Let R and T represent two $IVIF_S$'s. Consider a mapping $S : IVIF_S(X) \times IVIF_S(X) \to [0, 1]$, where $S(R, T)$ represents a SM between R and T. For $S(R, T)$ to be considered a SM, it must satisfy the following conditions:

1. $0 \leq S(R, T) \leq 1$,
2. $S(R, T) = 1 \iff R = T$,
3. $S(R, T) = S(T, R)$,
4. If $R \subseteq T \subseteq V$, then $S(R, V) \leq S(R, T)$, and $S(R, V) \leq S(T, V)$.

3. An Overview of Existing Similarity Measures

In this part, we will look at several existing SMs. Consider $R = \{(x_i, [\mu_R^-(x_i), \mu_R^+(x_i)], [\nu_R^-(x_i), \nu_R^+(x_i)])|x_i \in X\}$ and $T = \{(x_i, [\mu_T^-(x_i), \mu_T^+(x_i)], [\nu_T^-(x_i), \nu_T^+(x_i)])|x_i \in X\}$ are two $IVIF_S$'s defned in $X = \{x_1, x_2, \cdots, x_n\}$. Then the following Formulas (1) - (6) are $IVIF_S$-based SMs:

Singh's Measure [22]

$$S_C(R, T) = \frac{1}{n} \sum_{i=1}^{n} \frac{\alpha_1 \beta_1 + \delta_1 \gamma_1}{\sqrt{\alpha_1^2 + \delta_1^2}\sqrt{\beta_1^2 + \gamma_1^2}}. \tag{1}$$

$\alpha_1 = (\mu_R^-(x_i) + \mu_R^+(x_i))$, $\beta_1 = (\mu_T^-(x_i) + \mu_T^+(x_i))$, $\delta_1 = (\nu_R^-(x_i) + \nu_R^+(x_i))$ and $\gamma_1 = (\nu_T^-(x_i) + \nu_T^+(x_i))$.

Advantages and Limitations of Singh's Measure: Singh was the first to uncover cosine similarity measures for $IVIF_S$'s. It didn't meet some axiomatic defintions of SMs and didn't include the middle and boundary points of the intervals in their methods.

Xu's Measure [50]

$$S_1(R, T) = 1 - \sqrt[p]{\frac{1}{4n} \sum_{i=1}^{n} (|\mu_R^-(x_i) - \mu_T^-(x_i)|^p + |\mu_R^+(x_i) - \mu_T^+(x_i)|^p + |\nu_R^-(x_i) - \nu_T^-(x_i)|^p + |\nu_R^+(x_i) - \nu_T^+(x_i)|^p)}. \tag{2}$$

$$S_2(R, T) = 1 - \sqrt[p]{\frac{1}{n} \sum_{i=1}^{n} \max\left\{|\mu_R^-(x_i) - \mu_T^-(x_i)|^p, |\mu_R^+(x_i) - \mu_T^+(x_i)|^p, |\nu_R^-(x_i) - \nu_T^-(x_i)|^p, |\nu_R^+(x_i) - \nu_T^+(x_i)|^p\right\}}. \tag{3}$$

Advantages and Limitations of Xu's Measure: The authors developed distance and similarity measures of $IVIF_S$ on the basis of the Hamming, Euclidean and Hausdorff distance eventhough it didn't meet some axiomatic defintions of SMs.

Wei's Measure [49]

$$S_W(R,T) = \frac{1}{n}\sum_{i=1}^{n} \frac{2 - \min(\mu_i^-, \nu_i^-) - \min(\mu_i^+, \nu_i^+)}{2 + \max(\mu_i^-, \nu_i^-) + \max(\mu_i^+, \nu_i^+)}, \tag{4}$$

where $\mu_i^- = |\mu_R^-(x_i) - \mu_T^-(x_i)|$, $\mu_i^+ = |\mu_R^+(x_i) - \mu_T^+(x_i)|$, $\nu_i^- = |\nu_R^-(x_i) - \nu_T^-(x_i)|$, $\nu_i^+ = |\nu_R^+(x_i) - \nu_T^+(x_i)|$.

Advantages and Limitations of Wei's Measure: The experts developed the similarity measures using entropy measures for $IVIF_S$ although it didn't meet some axiomatic defintions of SMs.

Dhivya's Measure [48]

$$S_D(R,T) = 1 - \frac{1}{n}\sum_{i=1}^{n}\Big\{(\frac{1}{2}|\chi_R^-(x_i) - \chi_T^-(x_i)| + |\chi_R^+(x_i) - \chi_T^+(x_i)|)(1 - \frac{\sigma_R(x_i) + \sigma_T(x_i)}{2}) + |\sigma_R(x_i) + \sigma_T(x_i)|(\frac{\sigma_R(x_i) + \sigma_T(x_i)}{2})\Big\}, \tag{5}$$

where $\chi_R^- = \frac{(\mu_R^-(x_i)+1-\nu_R^-(x_i))}{2}$,
$\chi_R^+ = \frac{(\mu_R^+(x_i)+1-\nu_R^+(x_i))}{2}$,
$\chi_T^- = \frac{(\mu_T^-(x_i)+1-\nu_T^-(x_i))}{2}$
$\chi_T^+ = \frac{(\mu_T^+(x_i)+1-\nu_T^+(x_i))}{2}$,
$\sigma_R(x_i) = 1 - \frac{1}{2}(\mu_R^-(x_i) + \mu_R^+(x_i) + \nu_R^-(x_i) + \nu_R^+(x_i))$,
$\sigma_T(x_i) = 1 - \frac{(\mu_T^-(x_i)+\mu_T^+(x_i)+\nu_T^-(x_i)+\nu_T^+(x_i))}{2}$.

Advantages and Limitations of Dhivya's Measure: This article investigated effectively the similarity measures between $IVIF_S$ based on the mid points of transformed triangular fuzzy numbers. Lastly, it did not satisfy some axiomatic definitions of SMs.

Luo's Measure [47]

$$S_L(R,T) = 1 - \{\frac{1}{2n}\sum_{i=1}^{n} \frac{|t_1(\mu_R^-(x_i)-\mu_T^-(x_i))+(\mu_R^+(x_i)-\mu_T^+(x_i))-(\nu_R^-(x_i)-\nu_T^-(x_i))+(\nu_R^+(x_i)-\nu_T^+(x_i))|^p}{|2(t_1+1)|^p}$$

$$\frac{|t_2(\nu_R^-(x_i)-\nu_T^-(x_i))+(\nu_R^+(x_i)-\nu_T^+(x_i))-(\mu_R^-(x_i)-\mu_T^-(x_i))+(\mu_R^+(x_i)-\mu_T^+(x_i))|^p}{|2(t_2+1)|^p}\}^{\frac{1}{p}}, \tag{6}$$

where $t_1, t_2, p \in [1, +\infty)$, p is the L_p-norm.

Advantages and Limitations of Luo's Measure: Luo and Liang successfully examined the SMs between $IVIF_S$ based on the transformed interval-valued intuitionistic triangle fuzzy numbers, yet it did not satisfy some axiomatic definitions of SMs.

4. Development of New Similarity Measure

This section presents a novel mechanism for formulating a new $IVIF_S$ SM by modifying an existing $IVIF_S$ measure. The article by Singh [22] introduced a new SM for the comparison of $IVIF_S$'s. Particularly, it promotes new SM of $IVIF_S$'s. Next, the suggested study's flowchart is shown in Fig. 2.

Let R be an $IVIF_S$ in the universe of discourse $X = x$. The MF and NMF of $IVIF_S$ are described by $[\mu_R^-(x_i), \mu_R^+(x_i)]$, $[\nu_R^-(x_i), \nu_R^+(x_i)]$, which can be thought of as a vector representation containing the two elements. Accordingly, a cosine similarity measure equivalent to the one based on Bhattacharya's distance is provided for $IVIF_S$'s.

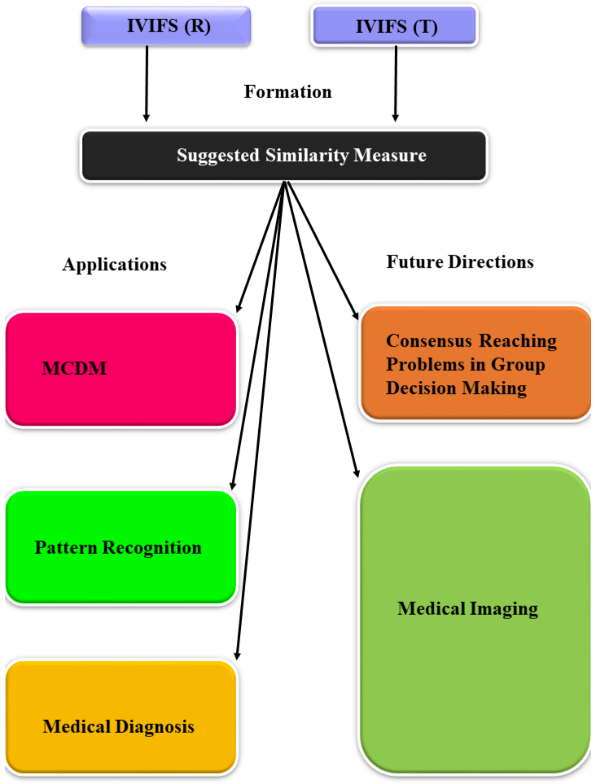

Fig. 2. the suggested study's flowchart

Definition 4.1.

Consider $R=\{(x_i,[\mu_R^-(x_i),\mu_R^+(x_i)],[\nu_R^-(x_i),\nu_R^+(x_i)])|x_i\in X\}$ *and* $T=\{(x_i,[\mu_T^-(x_i),\mu_T^+(x_i)],[\nu_T^-(x_i),\nu_T^+(x_i)])|x_i\in X\}$ are two $IVIF_S$'s defned in $X=\{x_1,x_2,\cdots,x_n\}$, the novel SM among the $IVIF_S$'s R and T ($S_{PP}(R,T)$) is specifed as follows:

$$S_{PP}(R,T)=\frac{1}{3}\left(S^1_{PP}(R,T)+S^2_{PP}(R,T)+S^3_{PP}(R,T)\right).$$

$$S_{PP}(R,T)=\frac{1}{3}\left(\frac{1}{n}\sum_{i=1}^n\frac{\alpha_1\beta_1+\delta_1\gamma_1}{\sqrt{\alpha_1^2+\delta_1^2}\sqrt{\beta_1^2+\gamma_1^2}}+\frac{1}{n}\sum_{i=1}^n\frac{\alpha_2\beta_2+\delta_2\gamma_2}{\sqrt{\alpha_2^2+\delta_2^2}\sqrt{\beta_2^2+\gamma_2^2}}=\frac{1}{n}\sum_{i=1}^n\frac{\alpha_3\beta_3+\delta_3\gamma_3}{\sqrt{\alpha_3^2+\delta_3^2}\sqrt{\beta_3^2+\gamma_3^2}}\right). \tag{7}$$

In this context, $S^1_{PP}(R,T)$ is the known SM [22], $S^2_{PP}(R,T)$ means the middle term of the $S^1_{PP}(R,T)$. $S^3_{PP}(R,T)$ signifies the complement of MF and NMF of R^-, R^+, T^- and T^+ respectively. In this measure, $\alpha_1=(\mu_R^-(x_i)+\mu_R^+(x_i))$, $\beta_1=(\mu_T^-(x_i)+\mu_T^+(x_i))$, $\delta_1=(\nu_R^-(x_i)+\nu_R^+(x_i))$, $\gamma_1=(\nu_T^-(x_i)+\nu_T^+(x_i))$, $\alpha_2=(\frac{\mu_R^-(x_i)+1-\nu_R^-(x_i)}{2}+\frac{\mu_R^+(x_i)+1-\nu_R^+(x_i)}{2})$, $\beta_2=(\frac{\mu_T^-(x_i)+1-\nu_T^-(x_i)}{2}+\frac{\mu_T^+(x_i)+1-\nu_T^+(x_i)}{2})$, $\delta_2=(\nu_R^-(x_i)+\nu_R^+(x_i))$, $\gamma_2=(\nu_T^-(x_i)+\nu_T^+(x_i))$, $\alpha_3=((1-\mu_R^-(x_i)+(1-\mu_T^-(x_i))$, $\beta_3=((1-\mu_R^+(x_i)+(1-\mu_T^+(x_i))$, $\delta_3=((1-\nu_R^+(x_i))+(1-\nu_T^+(x_i))$, and $\gamma_3=((1-\nu_R^-(x_i)+(1-\nu_T^-(x_i))$.

Additionally, the suggested SM fulfills the subsequent fundamental rules:

1. $0\leq S(R,T)\leq 1$,
2. $S(R,T)=1 \iff R=T$,
3. $S(R,T)=S(T,R)$,

4. If $R \subseteq T \subseteq V$, then $S(R,V) \leq S(R,T)$, and $S(R,V) \leq S(T,V)$.

Theorem 4.1. $0 \leq S_{PP}(R,T) \leq 1$.

Proof. According to the induction hypothesis, for any $\epsilon^1 \geq 0$, $\epsilon^2 \geq 0$, $\upsilon^1 \geq 0$, $\upsilon^2 \geq 0$, we have $(\epsilon^1\upsilon^2 - \epsilon^2\upsilon^1)^2 \geq 0 \iff (\epsilon^1\upsilon^1+\epsilon^2\upsilon^2)^2 \leq ((\epsilon^1)^2+(\epsilon^2)^2)((\upsilon^1)^2+(\upsilon^2)^2)$
iff $0 \leq (\epsilon^1\upsilon^1+\epsilon^2\upsilon^2) \leq \sqrt{(\epsilon^1)^2+(\epsilon^2)^2}\sqrt{(\upsilon^1)^2+(\upsilon^2)^2}$ iff $0 \leq \frac{\epsilon^1\upsilon^1+\epsilon^2\upsilon^2}{\sqrt{(\epsilon^1)^2+(\epsilon^2)^2}\sqrt{(\upsilon^1)^2+(\upsilon^2)^2}} \leq 1$.
R and T have MF and NMF intervals between 0 and 1. It is clear that $0 \leq S^1_{PP}(R,T) \leq 1$. Likewise, $0 \leq S^2_{PP}(R,T) \leq 1$ and $0 \leq S^3_{PP}(R,T) \leq 1$ are proven. Considering the fact, we get $0 \leq S_{PP}(R,T) \leq 1$. □

Theorem 4.2. $S_{PP}(R,T) = 1 \iff R = T$.

Proof. Now, let us examine the following induction, $S_{PP}(R,T) = 1 \iff (S^1_{PP}(R,T)+S^2_{PP}(R,T)+S^3_{PP}(R,T)) = 3$ if and only if $S^1_{PP}(R,T) = 1$, $S^2_{PP}(R,T) = 1$ and $S^3_{PP}(R,T) = 1 \Leftrightarrow$

$$[\mu^-_R(x_i) + \mu^+_R(x_i)] \times [\nu^-_T(x_i) + \nu^+_T(x_i)] = [\nu^-_R(x_i) + \nu^+_R(x_i)] \times [\mu^-_T(x_i) + \mu^+_T(x_i)]$$

$\Leftrightarrow$

$$[(\frac{\mu^-_R(x_i)+1-\nu^-_R(x_i)}{2})+(\frac{\mu^+_R(x_i)+1-\nu^+_R(x_i)}{2})]\times[\nu^-_T(x_i)+\nu^+_T(x_i)]=[\nu^-_R(x_i)+\nu^+_R(x_i)]\times[(\frac{\mu^-_T(x_i)+1-\nu^-_T(x_i)}{2})+(\frac{\mu^+_T(x_i)+1-\nu^+_T(x_i)}{2})]$$

$\Leftrightarrow$

$$[(1-\mu^-_R(x_i))+(1-\mu^-_T(x_i))]\times[(1-\nu^-_R(x_i))+(1-\nu^-_T(x_i))]=[(1-\nu^+_R(x_i))+(1-\nu^+_T(x_i))]\times[(1-\nu^-_R(x_i))+(1-\nu^-_T(x_i))] \Leftrightarrow$$

$$\begin{aligned}\mu^-_R(x_i) &= \mu^-_T(x_i),\\ \nu^-_R(x_i) &= \nu^-_T(x_i),\\ \mu^+_R(x_i) &= \mu^+_T(x_i),\\ \nu^+_R(x_i) &= \nu^+_T(x_i) \text{ if and only if } R = T.\end{aligned}$$

Since $S_{PP}(R,T) = \frac{1}{3}(S^1_{PP}(R,T) + S^2_{PP}(R,T) + S^3_{PP}(R,T))$, then $S_{PP}(R,T) = 1$ if and only if $R = T$. □

Theorem 4.3. $S_{PP}(R,T) = S_{PP}(T,R)$.

Proof. According to the principle of symmetry which states that, "Let R be a relation on A. Then, R is called symmetric, if $(a_1,a_2) \in R \Leftrightarrow (a_2,a_1) \in R$". Hence, we can conclude that $S^1_{PP}(R,T)$ is equal to $S^1_{PP}(T,R)$, $S^2_{PP}(R,T)$ is equal to $S^2_{PP}(T,R)$, and $S^3_{PP}(R,T)$ is equal to $S^3_{PP}(T,R)$. The equation $S_{PP}(R,T) = \frac{1}{3}(S^1_{PP}(R,T) + S^2_{PP}(R,T) + S^3_{PP}(R,T))$ implies that $S_{PP}(R,T)$ is equal to $S_{PP}(T,R)$. □

Theorem 4.4. $S_{PP}(R,V) \leq S_{PP}(R,T)$, and $S_{PP}(R,V) \leq S_{PP}(T,V)$ if $R \subseteq T \subseteq V$.

Proof. Consider the function h, defined as $h(\upsilon^1) = \frac{\epsilon^1\upsilon^1+\epsilon^2\upsilon^2}{\sqrt{(\upsilon^1)^2+(\upsilon^2)^2}}$. If we take h's derivative with regard to υ^1, we get $\frac{d}{d\upsilon^1}h(\upsilon^1) = \frac{\upsilon^2(\epsilon^1\upsilon^2-\epsilon^2\upsilon^1)}{((\upsilon^1)^2+(\upsilon^2)^2)^{\frac{3}{2}}}$, where $\epsilon^1 = [\mu^-_R(x_i) + \mu^+_R(x_i)]$, $\epsilon^2 = [\nu^-_R(x_i) + \nu^+_R(x_i)]$, $\upsilon^1 = [\mu^-_T(x_i) + \mu^+_T(x_i)]$, and $\upsilon^2 = [\nu^-_T(x_i) + \nu^+_T(x_i)]$. $h(\upsilon^1)$ is a decreasing function of υ^1 if $\frac{d}{d\upsilon^1}h(\upsilon^1) < 0$.

Consider the another function g, defined as $g(\upsilon^2) = \frac{\epsilon^1\upsilon^1+\epsilon^2\upsilon^2}{\sqrt{(\upsilon^1)^2+(\upsilon^2)^2}}$. If we take g's derivative with regard to υ^2, we get $\frac{d}{d\upsilon^2}g(\upsilon^2) = \frac{\upsilon^1(\epsilon^2\upsilon^1-\epsilon^1\upsilon^2)}{((\upsilon^1)^2+(\upsilon^2)^2)^{\frac{3}{2}}}$. $g(\upsilon^2)$ is an increasing function of υ^2 if $\frac{d}{d\upsilon^2}g(\upsilon^2) > 0$.

The finite universal set X has been divided into two disjoint subsets, X_1 & X_2, with the condition that $X_1 \cup X_2 = X$. For all $x_i \in X_1$,

$$(\mu_R^-(x_i)+\mu_R^+(x_i))\leq(\mu_T^-(x_i)+\mu_T^+(x_i))\leq(\mu_V^-(x_i)+\mu_V^+(x_i))\leq(\nu_V^-(x_i)+\nu_V^+(x_i))\leq(\nu_T^-(x_i)+\nu_T^+(x_i))\leq(\nu_R^-(x_i)+\nu_R^+(x_i)),$$

then for all $x_i \in X_2$,

$$(\mu_R^-(x_i)+\mu_R^+(x_i))\geq(\mu_T^-(x_i)+\mu_T^+(x_i))\geq(\mu_V^-(x_i)+\mu_V^+(x_i))\geq(\nu_V^-(x_i)+\nu_V^+(x_i))\geq(\nu_T^-(x_i)+\nu_T^+(x_i))\geq(\nu_R^-(x_i)+\nu_R^+(x_i)).$$

Therefore,

$$\begin{aligned} S^1_{PP}(R,V) &= \frac{E\Delta + BZ}{\sqrt{E^2+B^2}\sqrt{\Delta^2+Z^2}} \\ &\leq \frac{\alpha_1\beta_1+\delta_1\gamma_1}{\sqrt{\alpha_1^2+\delta_1^2}\sqrt{\beta_1^2+\gamma_1^2}} \\ &= S^1_{PP}(R,T) \end{aligned}$$

wherein $E=(\mu_R^-(x_i)+\mu_R^+(x_i))$, $\Delta=(\mu_T^-(x_i)+\mu_T^+(x_i))$, $B=(\nu_R^-(x_i)+\nu_R^+(x_i))$ and $Z=(\nu_T^-(x_i)+\nu_T^+(x_i))$. We can demonstrate that $S^1_{PP}(R,V)\leq S^1_{PP}(T,V)$. The forms of S^2_{PP} and S^3_{PP} are identical to S^1_{PP}. If $R\subseteq T\subseteq V$, we have $S^2_{PP}(R,V)\leq S^2_{PP}(R,T)$, $S^2_{PP}(R,V)\leq S^2_{PP}(T,V)$, and $S^3_{PP}(R,V)\leq S^3_{PP}(R,T)$, $S^3_{PP}(R,V)\leq S^3_{PP}(T,V)$. Therefore, $S_{PP}(R,V) \leq S_{PP}(R,T)$ and $S_{PP}(R,V) \leq S_{PP}(T,V)$ if $R \subseteq T \subseteq V$. Therefore, $S_{PP}(R,T)$ is a similarity measure. □

Remark 1. Consider the following two $IVIF_S$'s: $R=\{x,[0,0.1],[0,0.1]\}$ and $T=\{x,[0.1,0.3],[0,0.4]\}$. The CS_M for R and T is $S_C(R,T) = 1$. $R \neq T$, yet the similarity level is equal to 1. As a result, Definition (2.5)'s condition (ii) is not met. On the other hand, $S_{PP}(R,T) = 0.9783$ between $IVIF_S$'s R and T. As a result, it meets criterion (ii) of Definition (2.5).

Remark 2. If $R = \{x,[0,0],[0.1,0.2]\}$ and $T = \{x,[0.1,0.2],[0,0]\}$ are two $IVIF_S$'s, then the CS_M between them is zero. This result is unreasonable. After applying the proposed SM, the obtained result for $S_{PP}(R,T) = 0.6472$, which is considered reasonable.

Note 1. Based on the observations made in Remarks 1 and 2, it is demonstrated that S_{PP} is more effective and reasonable than CS_M between $IVIF_S$'s.

Example 4.1. If R_i and T_i are two $IVIF_S$'s, we can calculate the SM's between them using the various SM's represented in Table 1.

Table 1. A comparative analysis of SMs within the context of $IVIF_S$'s

	1	2	3	4
R_i	([0.2,0.3], [0.4,0.6])	([0.2,0.3], [0.4,0.6])	([0.2,0.3], [0.3,0.5])	([0.2,0.3], [0.3,0.5])
T_i	([0.3,0.4], [0.4,0.6])	([0.3,0.4], [0.3,0.5])	([0.3,0.4], [0.4,0.6])	([0.3,0.4], [0.3,0.5])
S_1 [50]	0.90	0.90	0.90	0.95
S_2 [50]	0.90	0.90	0.90	0.90
S_D [48]	1.00	0.98	0.95	0.94
S_L [47]	0.95	0.90	0.80	0.94
S_C [22]	0.98	0.96	0.99	0.98
S_{PP}	0.98	0.97	0.99	0.96

In Table 1, when we examine the first and second columns, we observe that $S_i(R_1,T_1)=S_i(R_2,T_2)$ $(i = 1, 2)$, given the condition $R_1 = R_2$ and $T_1 \neq T_2$. Similarly, upon comparing the third and fourth columns, we identify that $S_3(R_3,T_3) = S_4(R_4,T_4)$ when $R_3 = R_4$ and $T_3 \neq T_4$. Consequently, it becomes evident that the SM's S_1 and S_2 may not be suitable. In the meantime, we discover that $S_D(R_1,T_1) = 1$ when $R_1 \neq T_1$, which contradicts the second axiom of SM definition. Most crucially, the suggested SM's, S_{PP} and S_L, can effectively address these disadvantages.

It is also worth noting that previous studies [47], [23], [22] did not incorporate the middle and boundary points of the intervals into their approaches. But our proposed measure incorporates the middle and boundary terms as well as it qualifies the axiomatic definitions. As indicated in Table 1, our proposed measure consistently outperforms other sophisticated techniques. Consequently, we can conclude that our innovative $IVIF_S$ SM is more reasonable than the alternatives.

5. Applications

In this part, real-world issues are solved using the suggested SM in an $IVIF_S$'s environment, and the outcomes are compared with some of the other SMs that are currently in use. The proposed SM can be applied to real-world problems such as PR, MCDM and MD by representing data as $IVIF_S$'s.

5.1. Applications for PR

5.1.1. Procedure for PR

Taking X to be a finite universe of discourse, consisting of elements $x_1,x_2 \cdots ,x_n$, we can observe that there are m patterns labeled by $IVIF_S$'s $R_j=\{(x_1,[\mu^-_{R_j}(x_1),\mu^+_{R_j}(x_1)],[\nu^-_{R_j}(x_1),\nu^+_{R_j}(x_1)]), \cdots ,(x_n,[\mu^-_{R_j}(x_n),\mu^+_{R_j}(x_n)],[\nu^-_{R_j}(x_n),\nu^+_{R_j}(x_n)]):x_1,\cdots ,x_n \in X\}$ (where j ranges from 1 to m). Additionally, there is a test sample that has been classified with an $IVIF_S$ $T=\{(x_1,[\mu^-_T(x_1),\mu^+_T(x_1)],[\nu^-_T(x_1),\nu^+_T(x_1)]), \cdots ,(x_n,[\mu^-_T(x_n),\mu^+_T(x_n)],[\nu^-_T(x_n),\nu^+_T(x_n)]):x_1,\cdots ,x_n \in X\}$. The recognition procedure can be described as given:

Step 1: Calculate the $S(R_j, T)$ similarity measure for R_j (j lies between 1 to m) & T.

Step 2: To find the highest $S(R_{j_0}, T)$ value, we need to compare the values of $S(R_j, T)$ for each j (where j ranges from 1 to m). In other words, we want to determine the maximum value among $S(R_1, T), \cdots , S(R_m, T)$. Afterwards, the test sample T is classified according to the pattern R_{j_0}.

5.1.2. Ore Classification in a Coal Mining Area

In PR, the SM can help in comparing patterns or objects represented by $IVIF_S$'s, where objects may have uncertain boundaries or characteristics.

Example 5.1. In a coal mining area, there are four different types of ores labeled R_j (j=1,2,3,4). Each type of ore is represented by $IVIF_S$'s

$$R_j=\{(x_1,[\mu^-_{R_j}(x_1),\mu^+_{R_j}(x_1)],[\nu^-_{R_j}(x_1),\nu^+_{R_j}(x_1)]),...,(x_4,[\mu^-_{R_j}(x_4),\mu^+_{R_j}(x_4)],[\nu^-_{R_j}(x_4),\nu^+_{R_j}(x_4)]):x_1,...,x_4 \in X\},$$

as shown in Table 2. Our objective is to simply classify T into one of the four categories of ores mentioned above with the help of SM, as T is an unknown ore. Calculate the $S(R_j, T)$ similarity between R_j and T.

Table 2. Feature matrices

	F-1	F- 2	F-3	F- 4
R_1	([0.10, 0.50], [0.20, 0.30])	([0.10, 0.30], [0.00, 0.20])	([0.30, 0.50], [0.20, 0.40])	([0.20, 0.50], [0.10, 0.30])
R_2	([0.20, 0.40], [0.15, 0.35])	([0.20, 0.20], [0.05, 0.15])	([0.20, 0.60], [0.30, 0.30])	([0.30, 0.40], [0.15, 0.25])
R_3	([0.15, 0.30], [0.30, 0.40])	([0.20, 0.40], [0.50, 0.60])	([0.50, 0.60], [0.15, 0.35])	([0.25, 0.45], [0.30, 0.40])
R_4	([0.20, 0.35], [0.10, 0.65])	([0.35, 0.60], [0.05, 0.30])	([0.15, 0.30], [0.40, 0.55])	([0.15, 0.25], [0.45, 0.55])
T	([0.30, 0.40], [0.10, 0.50])	([0.10, 0.40], [0.25, 0.40])	([0.20, 0.30], [0.10, 0.35])	([0.15, 0.40], [0.20.0.50])

Looking at the results in Table 3, it is evident that when using S_1 for PR, the authors find that $S_1(R_1,T) = S_1(R_2,T) = S_1(R_4,T) > S_1(R_3,T)$. Thus, we cannot categorize sample T into a pattern.

$S_W(R_2,T) = S_W(R_4,T) > S_W(R_1,T) = S_W(R_3,T)$ can be obtained using S_W pattern recognition. Thus, we cannot determine if sample T belongs to R_2 or R_4.

Additionally, if we use S_D for PR, we find that $S_D(R_3,T) = S_D(R_4,T) > S_D(R_2,T) > S_D(R_1,T)$. This means we cannot determine whether sample T belongs to R_3 or R_4.

Moreover, when we use S_C for PR, we find that $S_C(R_4,T) < S_C(R_1,T) = S_C(R_2,T) < S_C(R_3,T)$. This means we cannot pinpoint whether sample T belongs to R_1 or R_2.

Likewise, by looking at Table 3, we can see that $S_L(R_1,T) > S_L(R_2,T) < S_L(R_3,T) > S_L(R_4,T)$. Additionally, when using S_{PP} for pattern recognition, we find that $S_{PP}(R_1,T) < S_{PP}(R_2,T) < S_{PP}(R_3,T) > S_{PP}(R_4,T)$.

According to the recognition principle, both S_2 and S_L yield similar recognition results, meaning that sample T can be categorized as pattern R_3. However, when using S_2 for similarity measurement, we cannot differentiate between R_2 and R_4. We conclude from Fig. 3 and Table 3 that S_L and S_{PP} measures yield mostly the same results, but S_{PP} is higher than S_L. In this way, pattern R_3 can be applied to sample T.

Table 3. PR outcomes using various SMs

	$S(R_1,T)$	$S(R_2,T)$	$S(R_3,T)$	$S(R_4,T)$	Classification results
S_1 [22]	0.87	0.87	0.86	0.87	N.A.
S_2 [22]	0.75	0.76	0.79	0.76	N.A.
S_W [49]	0.78	0.79	0.78	0.79	N.A.
S_D [48]	0.82	0.86	0.88	0.88	N.A.
S_L [47]	0.82	0.81	0.88	0.75	R_3
S_C [22]	0.95	0.95	0.97	0.92	N.A.
S_{PP}	0.95	0.96	0.97	0.93	R_3

N.A. not applicable

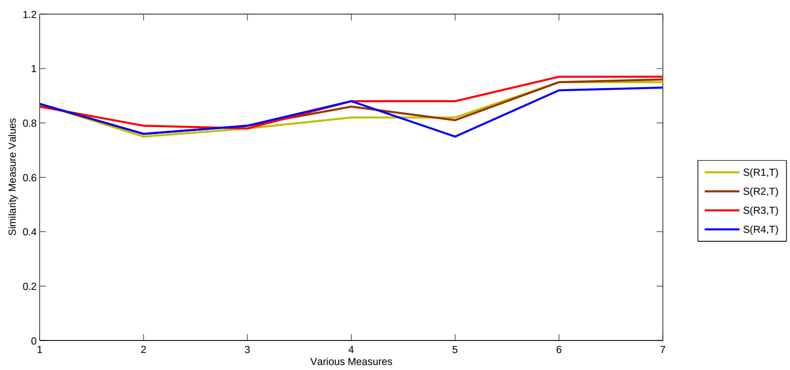

Fig. 3. Classification results of PR problem

Example 5.2. To illustrate the proposed SM, let us consider a PR scenario where we are trying to classify different building materials. In our feature space, represented by $X = \{x_1, x_2, \cdots, x_{12}\}$, there are four main classes of building materials denoted by $IVIF_S$

$$R_j = \{(x_1,[\mu^-_{R_j}(x_1),\mu^+_{R_j}(x_1)],[\nu^-_{R_j}(x_1),\nu^+_{R_j}(x_1)]),\cdots,(x_{12},[\mu^-_{R_j}(x_{12}),\mu^+_{R_j}(x_{12})],[\nu^-_{R_j}(x_{12}),\nu^+_{R_j}(x_{12})]) : x_1,\cdots,x_{12} \in X\}(j=1,\cdots,12).$$

Now, we have an unknown pattern T as all shown in Table 4. Next, we use the proposed formula to calculate the SM, $S(R_j, T)$, between the $IVIF_S$ $R_j (j = 1, 2, 3, 4)$ and T. This SM is a unique aspect of S_1 and S_2, and the computed result matches what is mentioned in the literature [47]. However, based on the recognition principle, as shown in Table 5 and Figs. 4, 5, we can accurately classify the unknown pattern in R_4 by using this SM. This result aligns with the findings of Luo and Liang in 2018.

Table 4. Unknown pattern T

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
R_1	([0.1,0.2],[0.5,0.6])	([0.1,0.2],[0.7,0.8])	([0.5,0.6],[0.3,0.4])	([0.8,0.9],[0.0,0.1])	([0.4,0.5],[0.3,0.4])	([0.0,0.1],[0.8,0.9])	([0.3,0.4],[0.5,0.6])	([1.0,1.0],[0.0,0.0])	([0.2,0.3],[0.6,0.7])	([0.4,0.5],[0.4,0.5])	([0.7,0.8],[0.1,0.2])	([0.4,0.5],[0.4,0.5])
R_2	([0.5,0.6],[0.3,0.4])	([0.6,0.7],[0.1,0.2])	([1.0,1.0],[0.0,0.0])	([0.1,0.2],[0.6,0.7])	([0.0,0.1],[0.8,0.9])	([0.7,0.8],[0.1,0.2])	([0.5,0.6],[0.3,0.4])	([0.6,0.7],[0.2,0.3])	([1.0,1.0],[0.0,0.0])	([0.1,0.2],[0.7,0.8])	([0.0,0.1],[0.8,0.9])	([0.7,0.8],[0.1,0.2])
R_3	([0.4,0.5],[0.3,0.4])	([0.6,0.7],[0.2,0.3])	([0.9,1.0],[0.0,0.0])	([0.0,0.1],[0.8,0.9])	([0.0,0.1],[0.8,0.9])	([0.6,0.7],[0.2,0.3])	([0.1,0.2],[0.7,0.8])	([0.2,0.3],[0.6,0.7])	([0.5,0.6],[0.2,0.4])	([1.0,1.0],[0.0,0.0])	([0.3,0.4],[0.4,0.5])	([0.0,0.1],[0.8,0.9])
R_4	([1.0,1.0],[0.0,0.0])	([1.0,1.0],[0.0,0.0])	([0.8,0.9],[0.0,0.1])	([0.7,0.8],[0.1,0.2])	([0.0,0.1],[0.7,0.9])	([0.0,0.1],[0.8,0.9])	([0.1,0.2],[0.7,0.8])	([0.1,0.2],[0.7,0.8])	([0.4,0.5],[0.3,0.4])	([1.0,1.0],[0.0,0.0])	([0.3,0.4],[0.4,0.5])	([0.0,0.1],[0.8,0.9])
T	([0.9,1.0],[0.0,0.0])	([0.9,1.0],[0.0,0.0])	([0.7,0.8],[0.1,0.2])	([0.6,0.7],[0.1,0.2])	([0.0,0.1],[0.8,0.9])	([0.1,0.2],[0.7,0.8])	([0.1,0.2],[0.7,0.8])	([0.1,0.2],[0.7,0.8])	([0.4,0.5],[0.3,0.4])	([1.0,1.0],[0.0,0.0])	([0.3,0.4],[0.4,0.5])	([0.0,0.1],[0.7,0.9])

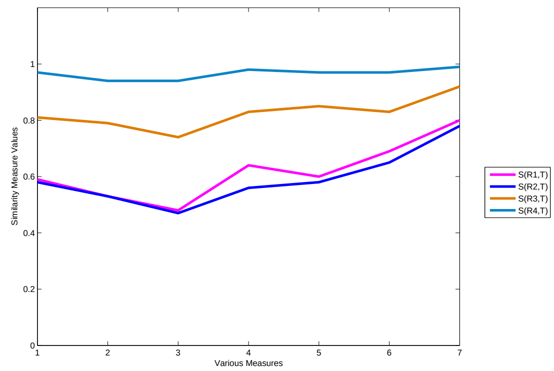

Fig. 4. Recognition results of PR problem

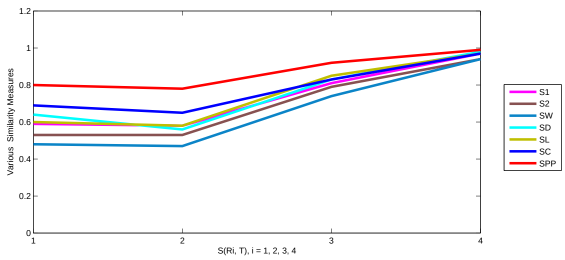

Fig. 5. Comparison graph of various similarity measures to PR problem

5.2. Application for MD

Medical diagnosis is the process of determining the cause of a person's symptoms or condition by analyzing their medical history, conducting physical examinations, and using diagnostic tests such as blood tests or imaging scans. It helps doctors identify and treat illnesses or diseases effectively.

Researchers have explored numerous approaches to tackle medical diagnosis problems, looking at them from various angles. In this particular approach, they have used pattern recognition algorithms. In this method, patients are treated like unknown test cases, diseases are treated as distinct

Table 5. PR outcomes using various SMs

	$S(R_1,T)$	$S(R_2,T)$	$S(R_3,T)$	$S(R_4,T)$	Recognition results
S_1 [50]	0.59	0.58	0.81	0.97	R_4
S_2 [50]	0.53	0.53	0.79	0.94	R_4
S_W [49]	0.48	0.47	0.74	0.94	R_4
S_D [48]	0.64	0.56	0.83	0.98	R_4
S_L [47]	0.60	0.58	0.85	0.97	R_4
S_C [22]	0.69	0.65	0.83	0.97	R_4
S_{PP}	0.80	0.78	0.92	0.99	R_4

patterns, and the set of symptoms serves as the set universe of discourse. The primary purpose is to classify patients and determine which disorder they might have.

Example 5.3. Let $X = \{x_1(\text{temperature}), x_2(\text{cough}), x_3(\text{headache}), x_4(\text{stomach pain})\}$ represent a set of symptoms, and let $R = \{R_1, R_2, R_3, R_4\}$ denote a set of diagnoses, where R_1= viral fever, R_2= typhoid, R_3= pneumonia, and R_4= stomach problem. Then the disease-symptom matrix with an $IVIF_S$ representation is listed in Table 6. Suppose a patient T, with respect to all the symptoms, can be represented by the following $IVIF$ information.

$$T=\{(x_1,[0.4,0.5],[0.1,0.2]),(x_2,[0.7,0.8],[0.1,0.2]),(x_3,[0.9,0.9],[0,0.1]),(x_4,[0.3,0.5],[0.2,0.4])\}.$$

Table 6. Symptom- disease relationship matrix

	x_1	x_2	x_3	x_4
R_1	([0.8,0.9],[0.0,0.1])	([0.7,0.8],[0.1,0.2])	([0.5,0.6],[0.2,0.3])	([0.6,0.8],[0.1,0.2])
R_2	([0.5,0.6],[0.1,0.3])	([0.8,0.9],[0.0,0.1])	([0.6,0.8],[0.1,0.2])	([0.4,0.6],[0.1,0.2])
R_3	([0.7,0.8],[0.1,0.2])	([0.7,0.9],[0.0,0.1])	([0.4,0.6],[0.2,0.4])	([0.3,0.5],[0.2,0.4])
R_4	([0.8,0.9],[0.0,0.1])	([0.7,0.8],[0.1,0.2])	([0.7,0.9],[0.0,0.1])	([0.8,0.9],[0.0,0.1])

Our main objective is to identify if patient T has any of the diseases R_1, R_2, R_3 and R_4. We will use $IVIF_S$'s, and we can see the expected results indicated in Table 7. These results are also presented graphically in Fig. 6.

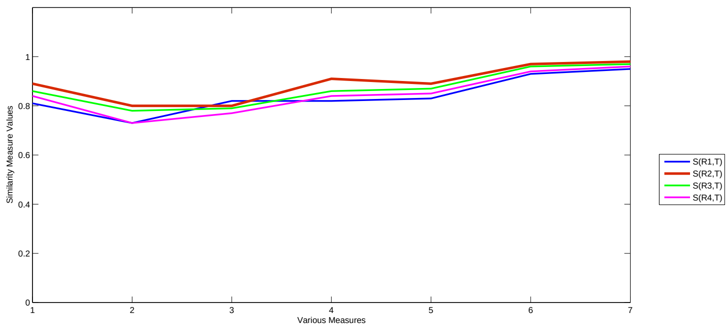

Fig. 6. Recognition results of medical diagnosis problem

Using the "maximum similarity principle,"we can observe that $S_{PP}(R_2,T)$ appears to be the highest. However, the SM S_2 could not clearly distinguish between R_1 and R_4. So, following the "recognition principle," we can say that patient T has typhoid. Finally, the performance of the suggested measure is shown in the previously described Table 7 and Figs. 6 and 7.

5.3. MCDM Problems

The process of finding and selecting choices based on the decision maker's values and preferences is known as decision making. The decision-making process is the selection of the best choice

Table 7. Computed values under different SMs

	$S(R_1,T)$	$S(R_2,T)$	$S(R_3,T)$	$S(R_4,T)$	Recognition results
S_1 [50]	0.81	0.89	0.86	0.84	R_2
S_2 [50]	0.73	0.80	0.78	0.73	R_2
S_W [49]	0.82	0.80	0.79	0.77	R_2
S_D [48]	0.82	0.91	0.86	0.84	R_2
S_L [47]	0.83	0.89	0.87	0.85	R_2
S_C [22]	0.93	0.97	0.96	0.94	R_2
S_{PP}	0.95	0.98	0.97	0.96	R_2

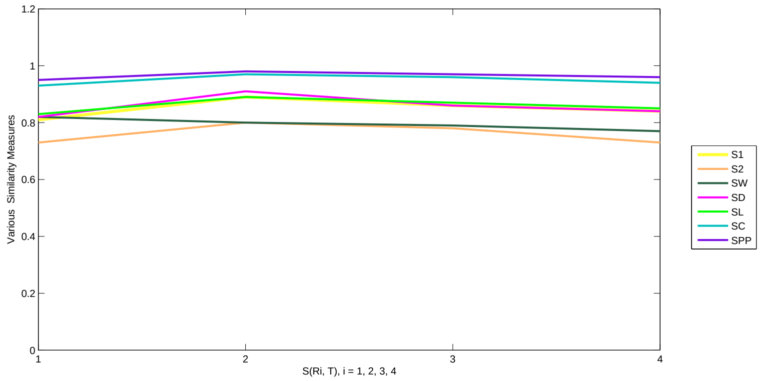

Fig. 7. Comparison graph of various SMs to medical diagnosis problem

from suitable alternatives. The proposed SM can be utilized to assess the similarity between alternatives described by $IVIF_S$'s in different criteria dimensions. This can assist decision-makers in ranking or selecting the most suitable alternatives in complex decision-making scenarios where uncertainty and imprecision are prevalent.

5.3.1. Optimizing Decisions: An Advanced MCDM Algorithm

In this subsection, we introduce a SM designed to address MCDM problems. We have a set of alternatives represented as $B = \{B_1, B_2, \cdots, B_m\}$, and a set of criteria represented as $C = \{C_1, C_2, \cdots, C_n\}$. Each alternative B_i for a criteria C is represented using $IVIF_S$ as given:

$$B_i = \left\{ (C_j, [\mu_{ij}^-, \mu_{ij}^+], [\nu_{ij}^-, \nu_{ij}^+]) : C_j \in C \right\}, i = 1, 2, \cdots, m.$$

With the help of the recommended measure 7, we are going to lay out an algorithm for solving MCDM problems.

Step 1: Establish the both positive (B^+) and negative (B^-) ideal solutions as follows:

$$B^+ = \{([\mu_{1+}^-, \mu_{1+}^+], [\nu_{1+}^-, \nu_{1+}^+]), ([\mu_{2+}^-, \mu_{2+}^+], [\nu_{2+}^-, \nu_{2+}^+]), \cdots ([\mu_{n+}^-, \mu_{n+}^+], [\nu_{n+}^-, \nu_{n+}^+])\}$$
$$B^- = \{([\mu_{1-}^-, \mu_{1-}^+], [\nu_{1-}^-, \nu_{1-}^+]), ([\mu_{2-}^-, \mu_{2-}^+], [\nu_{2-}^-, \nu_{2-}^+]), \cdots ([\mu_{n-}^-, \mu_{n-}^+], [\nu_{n-}^-, \nu_{n-}^+])\}$$

Here,

$$([\mu_{j+}^-, \mu_{j+}^+], [\nu_{j+}^-, \nu_{j+}^+]) = ([\max_i \mu_{ij}^-, \max_i \mu_{ij}^+], [\min_i \nu_{ij}^-, \min_i \nu_{ij}^+]),$$
$$([\mu_{j-}^-, \mu_{j-}^+], [\nu_{j-}^-, \nu_{j-}^+]) = ([\min_i \mu_{ij}^-, \min_i \mu_{ij}^+], [\max_i \nu_{ij}^-, \max_i \nu_{ij}^+]),$$

where $j = 1, 2, \cdots, n$.

Step 2: Use the following formula to calculate the degree of similarity for both positive $S(B_i, B^+)$ and negative $S(B_i, B^-)$ ideal solutions as follows :

$$S(B_i, B^+) = \frac{1}{3}(S^1_{PP}(B_i, B^+) + S^2_{PP}(B_i, B^+) + S^3_{PP}(B_i, B^+)) \tag{8}$$

$$S(B_i, B^-) = \frac{1}{3}(S^1_{PP}(B_i, B^-) + S^2_{PP}(B_i, B^-) + S^3_{PP}(B_i, B^-)) \tag{9}$$

Step 3: Apply Eqs. (8) - (9) to determine $S(B_i)$, the relative SM of B_i in regard to B^+ and B^-:

$$S(B_i) = \frac{S(B_i, B^+)}{S(B_i, B^-) + S(B_i, B^+)}, (i = 1, 2, \cdots, n). \tag{10}$$

Step 4: Choose the highest degree of similarity $S(B_t)$ from the set of $S(B_i)$ where i equals to $1, 2, \cdots, n$. In conclusion, $S(B_t)$ is the optimal choice.

5.3.2. MCDM Application

Example 5.4. A company needs to choose a distributor for their product. They are looking at six evaluation criteria: price (H_1), deadline (H_2), quality (H_3), level of technology (H_4), service (H_5), and the potential for future cooperation (H_6). They have five distributors to choose from, labeled B_i $(i = 1, 2, 3, 4, 5)$. Experts [49] have analyzed these suppliers using the criteria mentioned above, and here are the results they found in Table 8:

Table 8. Results of distributor

	H_1	H_2	H_3	H_4	H_5	H_6
B_1	([0.4,0.5],[0.2,0.3])	([0.6,0.8],[0.1,0.2])	([0.4,0.5],[0.2,0.4])	([0.8,0.9],[0.1,0.1])	([0.2,0.6],[0.2,0.3])	([0.5,0.7][0.1,0.2])
B_2	([0.5,0.7],[0.1,0.2])	([0.6,0.8],[0.1,0.2])	([0.3,0.4],[0.4,0.6])	([0.8,0.9],[0.0,0.1])	([0.2,0.5],[0.3,0.4])	([0.1,0.2][0.4,0.5])
B_3	([0.2,0.3],[0.6,0.7])	([0.4,0.5],[0.3,0.4])	([0.7,0.8],[0.1,0.2])	([0.2,0.5],[0.1,0.2])	([0.7,0.8],[0.0,0.1])	([0.5,0.6][0.2,0.4])
B_4	([0.5,0.6],[0.1,0.2])	([0.3,0.4],[0.2,0.3])	([0.5,0.8],[0.1,0.2])	([0.6,0.7],[0.1,0.2])	([0.3,0.4],[0.3,0.4])	([0.1,0.2][0.7,0.8])
B_5	([0.4,0.5],[0.3,0.4])	([0.8,0.9],[0.0,0.1])	([0.6,0.8],[0.1,0.2])	([0.8,0.9],[0.0,0.1])	([0.7,0.8],[0.1,0.2])	([0.5,0.6][0.1,0.2])

Using the preceding step 1, we obtain both the positive ideal solution B^+ and the negative ideal solution B^-, as shown in Table 9:

Table 9. The positive ideal solution B^+ and the negative ideal solution B^-

	H_1	H_2	H_3	H_4	H_5	H_6
B^+	([0.5,0.7],[0.1,0.2])	([0.8,0.9],[0.0,0.1])	([0.7,0.8],[0.1,0.2])	([0.8,0.9],[0.0,0.1])	([0.7,0.8],[0.0,0.1])	([0.5,0.7][0.1,0.2])
B^-	([0.2,0.3],[0.6,0.7])	([0.3,0.4],[0.3,0.4])	([0.3,0.4],[0.4,0.6])	([0.2,0.5],[0.1,0.2])	([0.2,0.4],[0.3,0.4])	([0.1,0.2][0.7,0.8])

We have used equations (8) - (9) to calculate SMs and decision outcomes, and you can see the results in Table 10 for comparison. We have also represented these findings graphically in Fig. 8. Notably, in Table 10, some rankings have stayed consistent, with B_5 emerging as the top choice. This highlights the effectiveness of our decision-making process using the recommended SM. Furthermore, most of the rankings in Table 10 closely resemble those seen in the specific example using $IVIF_S$. However, it's important to note that the way we rank using the suggested MCDM approach with $IVIF_S$ SMs in Table 10 differs from some of those depending on the MCDM method with $IVIF_S$ SMs in the same table. Therefore, its impact on the ranking of alternatives underscores its importance in the proposed MCDM applications.

While many advanced decision-making methods [47] - [50] in $IVIF_S$ settings haven't addressed MCDM issues with $IVIF_S$ data, these approaches lack the use of complementary and middle terms for MF and NMF degrees, which are important factors. However, as shown in Table 10 and Fig. 8,

Table 10. Ranking results

	Measure value	Ranking	The best one
S_1 [50]	$0.57, 0.49, 0.50, 0.45, 0.66$	$S(B_1) > S(B_2) < S(B_3) > S(B_4) < S(B_5)$	B_5
S_2 [50]	$0.52, 0.50, 0.49, 0.48, 0.56$	$S(B_1) > S(B_2) > S(B_3) > S(B_4) < S(B_5)$	B_5
S_W [49]	$0.55, 0.46, 0.53, 0.48, 0.64$	$S(B_1) > S(B_2) < S(B_3) > S(B_4) < S(B_5)$	B_5
S_D [48]	$0.54, 0.50, 0.49, 0.48, 0.58$	$S(B_1) > S(B_2) > S(B_3) > S(B_4) < S(B_5)$	B_5
S_L [47]	$0.50, 0.49, 0.50, 0.49, 0.50$	$S(B_1) > S(B_2) < S(B_3) > S(B_4) < S(B_5)$	B_5
S_C [22]	$0.54, 0.48, 0.51, 0.48, 0.56$	$S(B_1) > S(B_2) < S(B_3) > S(B_4) < S(B_5)$	B_5
S_{PP}	$0.53, 0.50, 0.51, 0.49, 0.54$	$S(B_1) > S(B_2) < S(B_3) > S(B_4) < S(B_5)$	B_5

our proposed MCDM framework using $IVIF_S$ SMs can be effectively applied to MCDM problems with $IVIF_S$ data. This enhances the decision results more valuable and realistic for practical decision-making.

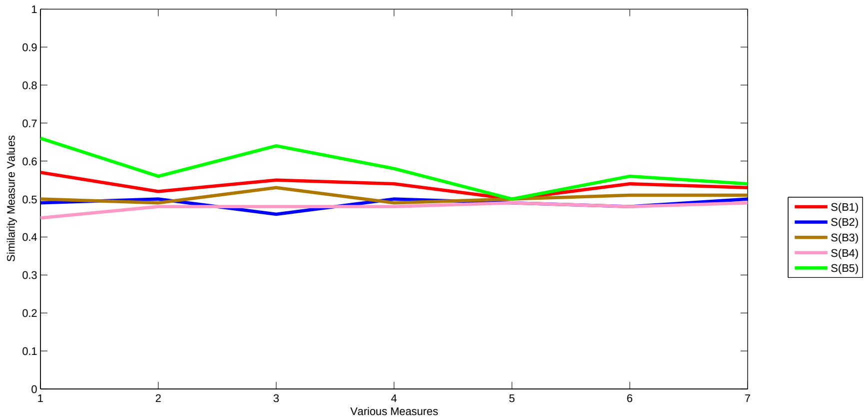

Fig. 8. Ranking results of MCDM problem

5.3.3. Results and Discussion

Certain similarity measures perform better in specific scenarios due to the reasons are the performance of provides the characteristics of the data, the requirements of the problem at hand, and the suitability of different measures for practical applications. These insights can guide algorithm design, model selection, and decision-making processes, ultimately leading to more effective solutions in real-world settings. Pattern recognition results can enhance real-world classification tasks by accurately identifying patterns in data, leading to improved decision-making and automation in various domains such as image recognition and fraud detection. Medical diagnosis can enhance healthcare decision-making by enabling more accurate diagnoses and personalized treatment plans, ultimately improving patient outcomes and resource allocation within healthcare systems.

Previous studies [47], [23], [22] did not consider the middle and boundary points of intervals in their methods. However, our proposed similarity measure includes these points and adheres to established definitions. Our measure consistently outperforms other advanced techniques, as shown in Table 1. By following the principle of "maximum similarity,"our measure consistently ranks the highest. While existing methods in $IVIF_S$ settings [22], [48] - [50] haven't effectively addressed MCDM problems with $IVIF_S$ data, they overlook crucial factors like complementary and middle terms. Our proposed MCDM framework, as depicted in Table 10 and Fig. 8, successfully applies $IVIF_S$ similarity measures to MCDM issues, resulting in more realistic decision outcomes. Based on these findings, we can state that our similarity measure outperforms others, making it a more reasonable choice for decision-making tasks.

5.3.4. Advantages of the Suggested Approach

The advantages of utilizing an IVIFS-based SM include the following typical benefits:

- The utilization of $IVIF_S$ enhances the reliability assessment of MF and NMF degree intervals, thereby making the IF_S more trustworthy. Consequently, the suggested $IVIF_S$ incorporates a significantly greater amount of valuable information compared to traditional IF_S.
- This work solves MCDM problems, PR, and medical diagnosis using the suggested $IVIF_S$ SM, which improves medical, pattern, and decision-making outcomes and outperforms existing $IVIF_S$-based SMs.
- One advantage of employing this method is its apparent simplicity in calculating the similarity level. Additionally, the merits of this research encompass a greater resemblance to real-world scenarios.
- Furthermore, the core advantage of this approach lies in its utilization of MF, NMF, middle, and complement degrees for determining similarity. Consequently, our proposed method may offer greater accuracy compared to existing advanced procedures.

6. Conclusion and Future Directions

In this research, we have created an innovative method for assessing the similarity of $IVIF_S$'s. Building upon an existing CS_M, our study has introduced a novel IVIFCSM, highlighting its essential characteristics. Furthermore, we have successfully applied this newly established SM to address a diverse range of real-world challenges, including PR, medical diagnosis, and MCDM. To illustrate the practicality of our approach, we have provided numerical examples showcasing its application in each context. Notably, our proposed technique yields higher similarity values in all these applications compared to other SMs, aligning with the maximum similarity principle and thus providing more informative results.

However, it's worth noting that the proposed method does come with certain limitations. Specifically, the suggested SM is applicable in scenarios where degrees of MF and NMF can be expressed as numerical values with intervals. Nevertheless, in many real-world situations, linguistic variables are utilized to convey qualitative information. Regrettably, this particular SM is not suitable for application in a linguistic context. Therefore, further investigation is necessary to explore the adaptation of this SM to accommodate linguistic IVIF information.

In future research, exploring advanced approaches like probabilities, power averages, and moving averages could help overcome these limitations. Probabilistic methods can capture the uncertainty inherent in linguistic information by assigning probabilities to different linguistic terms. Power averages and moving averages offer alternative ways to aggregate linguistic IVIF information, allowing for more nuanced comparisons and decision-making. By integrating these advanced techniques, it may be possible to develop SMs that are more suitable for handling linguistic IVIF information in real-world applications.

These methods could be applied to various fields such as medical imaging, social network and large-scale decision-making, and consensus building. Consensus measures, in particular, are crucial in group decision-making, as specialists need to agree on a solution. We will also work on developing consensus measures based on our proposed SM and applying them to tackle challenges in social network and large-scale decision-making in uncertain situations. Building on previous research, we are excited to use our measure to address consensus-related issues in group decision-making involving $IVIF_S$'s [51], [52]. Further, we plan to implement the systematic approach outlined for developing an AI-powered windshield crack detection system. By following the steps of data collection, preprocessing, model selection, training, evaluation, deployment, user interface design, testing, optimization, and maintenance, we aim to create a robust and reliable system that enhances automotive safety and maintenance. Additionally, we intend to explore the adaptation of similar methodologies to accommodate linguistic IVIF information, leveraging advanced techniques such as probabilities, power averages, and moving averages to improve similarity measures and address real-world challenges effectively.

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