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Study on Viral Transmission Impact on Human Population Using Fractional Order Zika Virus Model

Dhanalakshmi Palanisami [a,1,*], Shrilekha Elango [b,2]

[a] Department of Applied Mathematics, Bharathiar University, Tamil Nadu, India
[b] Department of Applied Mathematics, Bharathiar University, Tamil Nadu, India
[1] viga_dhanasekar@yahoo.co.in; [2] shrilekha.appliedmaths@buc.edu.in
* Corresponding Author

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ABSTRACT

This work comprises the spread of Zika virus between humans and mosquitoes as a mathematical simulation under fractional order, which also incorporates the asymptotically infected human population. For determining the solution of the model the fuzzy Laplace transform technique is utilized. By combining fuzzy logic with the Laplace transform, we can analyze systems even when we lack precise information. Further, the sensitivity analysis is performed to validate the model. On top of that the population dynamics of both human and mosquito populations are discussed using numerical data and the graphical result of the model is presented. The main objective of this work is to study the dynamics of the Zika virus and to examine the effect of virus on humans when the transmission occurs between humans and from mosquitoes, under fractional order. The outcome of these comparisons suggests that even by reducing a minute fractional part of transmission through mosquitoes results in a greater reduction of Zika exposed population. The comparisons improve the understanding of fractional level transmission resulting in more effective drug administration to patients. The Hyers-Ulam stability method is a mathematical technique used to study the stability of functional equations. Eventually, Ulam Hyers and Ulam Hyers Rassias stability are employed to assess the stability of the proposed model.

1. Introduction

Zika virus has the potential to resurface as an epidemic once the COVID-19 pandemic ceases to exist. It is still active in tropical and subtropical places [1] and emerges as a serious risk, especially for pregnant mothers. The research [2] suggests that the Zika virus has been spreading in many Indian states, indicating the urgent need to enhance surveillance. Apart from mosquito bites, this virus can be spread through blood transfusion and sexual contact [3]. Around 2007, many cases were not reported and a maximum of 14 Zika virus cases were documented. On the Island of Yap, a significant Zika infection outbreak was observed in 2007 [4]. The WHO (World Health Organization) proclaimed a global epidemic in 2016 as a Public Health Emergency after an abundance of cases were reported in 27 American countries. Over one million Zika virus infections were recorded in Latin America between the years 2015 and 2016. Following that, the virus advanced quickly to 86 nations, with an aggregate of 223, 477 confirmed cases [5]. According to the Zika virus epidemiology update of 2022, around 40,249 Zika cases have been documented. Subsequently, the virus infection spread to southeast Asia, the Pacific Islands, French Polynesia, Tropical Africa, North, South and Central America. Zika virus has at least two extensive lineages namely, Asian and African lineages. In humans, the symptomatic infection may result in mild and self-limiting feverish disease, even though recent reports have advised a possible connection with additional serious sequelae [6]. The Zika virus is mostly caused by Aedes mosquitoes. In the areas of most occurrences of ZIKV (Zika Virus) infection, there is an increase in the cases of microcephaly. Also, the Zika virus affects foetal development on a large scale and leads to mild diseases in major infections. The incubation period for the Zika virus is 3-12 days, which is greater than the period of Chikungunya. It leads to mild fever, skin rashes, headache (retro-orbital) and Conjunctivitis. It also shows 20-25% of symptoms which is less when compared to Chikungunya and equal when compared to Dengue [7]. The severity of the Zika virus depends on various factors, such as a person's age, overall health, and immune response. In most cases, the Zika virus is mild and self-limiting, with symptoms lasting a few days to a week. Since the Zika virus is a vector-borne virus it leads to mild or no clinical symptoms. A branch of ecology known as population dynamics deals with changes in population size and density for different species across time and space. The first category involves quantitative analyses of variations in the size of populations and trends of increase or decrease for particular organisms, and investigations of the factors, physical and biological mechanisms underlying these variations. The second category, which addresses causal processes, is crucial as it can offer a broad framework for mosquito-controlling tactics. Forecasts of population patterns may also be improved with knowledge of the underlying causes guiding population dynamics [9].

Fractional derivatives can be applied in formulating many phenomena and in several scientific domains. This is due to the fact that accurate modeling of physical events requires both past and present time histories. The noteworthy fact of the fractional differential equation is its generalisation and hereditary property. This fact allows the flexible usage of fractional differential equations in mathematical modeling. Various derivatives exist such as Riemann-Liouville, Modified R-L (Riemann-Liouville), Caputo, Caputo-Fabrizio in fractional order derivative, conformable fractional derivative, etc., Fuzzy fractional calculus is introduced as a result of the combination of the aforementioned derivatives with fuzzy differences and/or fuzzy derivatives [14]. In 2010, the concept of fuzzy calculus in fractional order originated. In [12], the author utilised the Krasnoselskii-Krein type condition in solving the fuzzy equation. Through the technique of fuzzy fractional calculus, a financial system was modelled and investigated by the authors [15] and [39].

Mathematical modelling is employed to enhance our knowledge on the epidemiology of the disease and its management. In the process of solving the real world problem our prior aim is to formulate a mathematical model that would represent essential aspects of the current situation. To provide accurate predictions, a model's assumptions must coincide with reality, but since all models are approximations of reality, this connection is never perfect. A key advantage of mathematical modeling is that it lets us compare the model outcomes to evaluate our knowledge of the disease epidemiology and also aids in decision-making by projecting crucial issues in disease transmission [9]. Numerous mathematicians and biologists are interested in the study of the dynamics of the disease in recent years [10]-[11]. The fractional order of the mathematical model represents the natural actuality of the phenomena in a systematic approach. Fractional calculus is currently advancing, fast developing and set footprints in all scientific fields. In [13], the author has provided a detailed view of the future direction of research on fractional calculus. Mathematical modeling of the dynamical system has been advancing at a fast pace and has inspired many researchers of various fields to combine this concept in their field. Many studies on modelling the biological system under fractional order have been carried out to obtain a more comprehensive framework to analyze and understand the complex dynamics and behaviors observed in living organisms. Similarly, the authors Dhanalakshmi et al., [34] employed fractional derivatives in a tumor system along with the chemotherapy drugs and analysed the stability of the model under the Lyapunov concept. The authors Akindeinde et al., [29] presented an epidemic model for COVID-19 which is represented in both the integer and fractional order. The qualitative analysis, well posedness and global stability of the model were discussed in the above work.

Fuzzy fractional calculus or fuzzy fractional modeling is a newly emerging and rapidly developing research that combines fuzzy logic and fractional calculus. While it may not have received as much attention as fractional calculus or fuzzy logic on its own, the idea has gained popularity among researchers in recent years. A fuzzy model on alcohol consumption with health risk parameters was proposed in [37] which suggested that peer influence plays a greater role in alcohol-related health risks. The author Mpeshe used the fuzzy SEIR model to study the amoebiasis infection and came to the conclusion that the condition would probably last for the first month before gradually reaching a state of disease-free equilibrium [38].

Most often, the Zika virus infects people through mosquito bites, and it causes many secondary problems. Several experts have explored the dynamics of the Zika virus from various perspectives. For example, Zika infection analyzed by some mathematical models under ordinary differential equations is covered in [16] - [19]. The transmission of the Zika virus through sexual contact has been investigated in [16]. The transmission dynamics of Zika virus in Columbia were mathematically modeled in [17]. A mathematical Zika virus model with optimal control was presented in [18] and the global sensitivity was analysed using statistical techniques. Zika virus along with its mutations and the optimal controls are modeled into a mathematical system as [19]. In [20], the potential routes of virus transmission are investigated to provide an optimal solution. In [26], Khan et al. devised a dynamic framework that includes an asymptomatic Zika virus carrier as well as optimal control. The author also presented the model's findings using and without control, and analysed its stability. Furthermore, the author established the graphical conclusion for the model using the backward Runge-Kutta technique. The limitation of utilizing classical calculus in modeling is that it does not exhibit a memory effect. Modeling a concept that is dependent on prior behavior will be a challenge. A mathematical framework for the mosquito-borne Zika virus was developed under fractional order which can overcome the constraints of classical calculus. In [25], the author Ali et al., developed a Zika viral model with its mutation in a fractional environment and used a generalized predictor-corrector method to solve the numerical example. One of the best control methods for the Zika virus is creating awareness about it. A Zika virus model of fractional order was constructed and its long-term results were examined in [21] and the model has a close conclusion to the numerically solved system using Lagrange's interpolation polynomial. Numerically, the authors of [36] solved the fractional order of nonlinear systems and analysed the local and global stability. Laplace transform is known for its easy computations, that is, converting the differential equations to algebraic equations. By employing the algebraic techniques, the complex equations can be solved without large and complicated calculations.

The theory of Ulam Hyers stability has implications in various areas such as fluid flow, automatic control, biology, and mechanics, especially where it is challenging to find explicit solutions, and has drawn the interest of numerous researchers in recent years. In the work [40], the authors examined the tumor system and applied Ulam Hyers Stability to determine the stability of the system. The possible hindrance that may affect the precise outcome of the model is data limitation which can be surpassed by verifying the credibility of data with numerous sources. The quality of a model is determined by the data used to parameterize it. The model's data can vary depending on things like demographics, climate, nutrition, lifestyle, etc., which may modify the outcome. The key benefit of applying Ulam Hyers Stability is that finding the exact solution is not a necessary requirement. Ulam Hyers and Ulam Hyers Rassias Stability are implemented to examine the system's stability.

Motivated by the aforementioned initiatives, this paper focuses on the Zika virus model under a fractional environment and its stability analysis. The research contribution of this work is utilizing fuzzy fractional calculus in modeling and analysing the transmission of Zika virus by humans and mosquitoes. The Laplace transforms under fuzzy concept is utilized to solve the complex FDEs into an algebraic equation. Due to its significant role, it is useful in obtaining the solution of the system and proving its stability. Ulam Hyers and Ulam Hyers Rassias Stability is free from complex as well as elaborate calculations. This work investigates the impact of viruses on human populations when transmission happens between humans and mosquitoes in fractional order.

The following are the highlights of the work:

1. The Zika virus model is formulated by taking various parameters such as the awareness rate of the host, asymptotically infected carrier, etc., into consideration under fractional order.
2. Fuzzy Laplace transform is used in finding the solution's existence and its uniqueness.
3. Basic Reproduction number is determined and the sensitivity analysis on the model is performed.
4. The model is solved numerically for population dynamics and the solution is presented graphically.
5. The impact of viral transmission from human to human and mosquitoes to humans are examined separately and compared numerically as well as graphically.
6. The proposed model's stability is determined by Ulam Hyers and Ulam Hyers Rassias stability.

The overall structure of the paper is organised as follows: In section 2 the preliminaries and the essential properties are included. The model is formulated and the parameters are described in section 3. Section 4 comprises the properties of solution and analysis of the model. The section 5 involves numerical simulation and graphical framework of the model. Section 7 comprises the discussion of stability using Ulam Hyers and Ulam Hyers Rassias stability.

2. Preliminaries

This part of the article comprises the preliminaries and results of fractional calculus and laplace transform. The further properties and remarks on fractional differential equations can be referred to [32] and [33].

Definition 1. *The Caputo derivative is characterized as a fractional derivative which incorporates an integer-order ordinary derivative with a fractional integral [29]. It enables the differentiation of a function to a non-integer order with respect to time or another independent variable. The fractional order function z(t) under Caputo derivative is provided as follows,*

$$^{C}D^{\alpha}z(t) = \frac{1}{\Gamma(p-\alpha)}\int_a^t (s-t)^{p-\alpha-1}D^p z(s)ds, \quad p<\alpha<p-1, \quad t>a, \quad p\in\mathbb{N}$$

in which the function $z:[a,b]\to\mathbb{R}$ and $D^p z(s)$, $\forall$ p, are integrable. The mathematical description presented below denotes the gamma function $\Gamma(\alpha)$

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}e^{-t}dt \tag{1}$$

Definition 2. *The concept of Riemann-Liouville integration extends the concept of integration to non-integer orders [29]. The fractional order z(t) is Riemann-Liouville integrable for $\alpha>0$ if*

$$I_a^{\alpha}z(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-s)^{\alpha-1}z(s)ds, \quad I_a^0 z(t) = z(t).$$

in which the notation Γ denotes Euler gamma function.

Few properties related to the fractional calculus are presented here,

1. ${}^{C}D^{\alpha}X(t) = I_a^{k-\alpha}D^kX(s)ds$,
2. $I_a^{\alpha C}D^{\alpha}X(t) = X(t) - X(a)$ for $0 < \alpha < 1$.

Definition 3. *Let $\mathbb{R}$ denote the collection of real numbers [30]. A mapping $\mathtt{x} : \mathbb{R} \to [0, 1]$ is denoted as fuzzy number when it adheres to the below requirements,*

- $\mathtt{x}$ *is fuzzy convex, i.e.,* $\mathtt{x}(\lambda u + (1 - \lambda v)) \geq min[\mathtt{x}(u), \mathtt{x}(v)]$.
- $\mathtt{x}$ *is normal.*
- $supp(\mathtt{x}) = u \in \mathcal{R}|\mathtt{x}(u) > 0$ *denotes the support of* $\mathtt{x}$ *and its closure must be compact.*
- $\mathtt{x}$ *is upper semi-continuous.*

The $\mathcal{E}$ is referred to as the collection of all the convex, upper semi-continuous and normal fuzzy numbers. The r - level set is a bounded and closed interval $[\underline{h}(r), \overline{h}(r)]$ defined as

$$[h]^r = x : h(x) \geq r, r \in (0, 1],$$

where the closed and bounded interval is presented by $h = [\underline{h}(x), \overline{h}(x)] \in \mathcal{E}$.

Definition 4. *Let $\mathfrak{D}$ be a mapping such that $\mathcal{E} \times \mathcal{E} \to \mathcal{R}$ and f, h be two fuzzy number denoted by their parametric form as, $g = [\underline{f}(u), \overline{f}(u)]$ and $h = [\underline{h}(u), \overline{h}(u)]$ respectively [31]. The Hausdorff distance of g and h is given as,*

$$\mathfrak{D}(f, h) = \sup_{u \in [0,1]} [max|\underline{f}(u) - \underline{h}(u)|, |\overline{f}(u) - \overline{h}(u)|] \tag{2}$$

In the space $\mathcal{E}$, the metric $\mathfrak{D}$ satisfies the below conditions,

- $\mathfrak{D}(f\tau, h\tau) = |\tau|\mathfrak{D}(f, h)\forall f, h \in \mathcal{E}, \tau \in \mathbb{R}$
- $\mathfrak{D}(f + \tau_1, h + \tau_2) \leq \mathfrak{D}(f, h)\mathfrak{D}(\tau_1, \tau_2)\forall f, h, \tau_1, \tau_1 \in \mathcal{E}$
- $\mathfrak{D}(f + \omega, h + \omega) = \mathfrak{D}(f, h)\forall f, h, \omega \in \mathcal{E}$
- $(\mathcal{E}, \mathfrak{D})$ *is a complete metric space.*

Definition 5. *Consider $p, q \in \mathcal{E}$ and let $j \in \mathcal{E}$ such that $p = q + j$, then j is represented as H-difference of p and q which is defined by p ⊖ q. The Hukuhara difference is a mathematical technique used in the study of fuzzy sets. It is an extension of the traditional fuzzy set difference technique.*

Laplace transform is an efficient method and has easy computations for solving the fractional equation. Further, it is the generalisation of classical laplace transform. It has a great contribution in simplifying the complicated FDE into an algebraic equation. As a result, it serves as a useful tool for handling real-world problems.

Definition 6. *The formula given below provides the laplace transform of p(t) [25]:*

$$L[p(t)] = P(s) = \int_0^{\infty} e^{-st} \odot p(t)dt$$

$$L[p(t)] = \lim_{\tau \to \infty} \int_0^{\tau} e^{-st} \odot p(t)dt \tag{3}$$

where the laplace transform of p(t) is given as P(s). The integral diverges if the limit is removed and if the limit exists, the integral converges.

The level-wise form of a fuzzy number $p(t)$ under laplace transform is provided as,

$$L[p(t)] = P(s,r) = [\underline{P}(s,r), \overline{P}(s,r)]$$

in a bounded and closed interval $0 < r < 1$. The Caputo fractional differential equation under laplace transform becomes,

$$s^{\alpha} P(s) \ominus \sum_{k=0}^{n-1} s^{\alpha-k-1} \odot p^{k}(0) = L[{}^{C}D^{\alpha}(p(t))], \tag{4}$$

where the laplace transform of p(t) is given as P(s).

3. Model Formultion

Zika virus is mostly transmitted by mosquitoes and humans. In this model, the population of humans has been broken down into four groups susceptible $\mathtt{S}_{(\mathtt{h})}$, exposed $\mathtt{E}_{(\mathtt{h})}$, infected $\mathtt{I}_{(\mathtt{h})}$, asymptomatically infected $\mathcal{A}\mathtt{I}_{(\mathtt{h})}$ and recovered $\mathtt{R}_{(\mathtt{h})}$. Further, the population of mosquitoes is further classified into susceptible $\mathtt{S}_{(\mathtt{m})}$, exposed $\mathtt{E}_{(\mathtt{m})}$ and infected $\mathtt{I}_{(\mathtt{m})}$. Thus, the fractional order Zika virus model is formulated as:

$$\begin{aligned}
{}^{C}D^{\alpha}\mathtt{S}_{(\mathtt{h})} &= \Xi_{H} + \lambda\mathtt{R}_{(\mathtt{h})} - \beta_1\mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{h})} - \beta_2\mathtt{I}_{(\mathtt{m})}\mathtt{S}_{(\mathtt{h})} - (\omega_1+\kappa_1)\mathtt{S}_{(\mathtt{h})},\\
{}^{C}D^{\alpha}\mathtt{E}_{(\mathtt{h})} &= \beta_1\mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{h})} + \beta_2\mathtt{I}_{(\mathtt{m})}\mathtt{S}_{(\mathtt{h})} - (\kappa_1+\sigma_1)\mathtt{E}_{(\mathtt{h})},\\
{}^{C}D^{\alpha}\mathtt{I}_{(\mathtt{h})} &= \sigma_1\mathtt{E}_{(\mathtt{h})} - (\kappa_1+\gamma)\mathtt{I}_{(\mathtt{h})},\\
{}^{C}D^{\alpha}\mathcal{A}\mathtt{I}_{(\mathtt{h})} &= \Psi(1-\Phi)\mathtt{E}_{(\mathtt{h})} - \kappa_1\mathcal{A}\mathtt{I}_{(\mathtt{h})},\\
{}^{C}D^{\alpha}\mathtt{R}_{(\mathtt{h})} &= \gamma\mathtt{I}_{(\mathtt{h})} - \kappa_1\mathtt{R}_{(\mathtt{h})} + \omega_1\mathtt{S}_{(\mathtt{h})} - \lambda\mathtt{R}_{(\mathtt{h})},\\
{}^{C}D^{\alpha}\mathtt{S}_{(\mathtt{m})} &= \Xi_{M} - \mu\mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{m})} - (\omega_2+\kappa_2)\mathtt{S}_{(\mathtt{m})},\\
{}^{C}D^{\alpha}\mathtt{E}_{(\mathtt{m})} &= \mu\mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{m})} - (\omega_2+\sigma_2+\kappa_2)\mathtt{E}_{(\mathtt{m})},\\
{}^{C}D^{\alpha}\mathtt{I}_{(\mathtt{m})} &= \sigma_2\mathtt{E}_{(\mathtt{m})} - (\omega_2+\kappa_2)\mathtt{I}_{(\mathtt{m})}.
\end{aligned} \tag{5}$$

from which ${}^{C}D^{\alpha}$ implies the Caputo differential equation of order $\alpha \in [0,1]$. Case studies and outbreak investigations give extensive information on specific cases, such as disease progression, transmission patterns, etc., The proposed model is formulated by referring to the models in [18], [25], [26]. The main objective for selecting the Zika virus spread in the form of a mathematical model is to get an understanding of its transmission dynamics. Further, with the help of the mathematical model, we can investigate the spread of the virus within a population, identifying the parameters that have a major impact on the transmission and evaluating the impact caused by the transmission. Another significance of the model is that the parameter for the asymptotically infected population is included to provide better insight into the transmission dynamics. The parameters are raised to the power of α, in order to consider the dimensional consistency. The parameter and the explanation of the model (5) are presented in the Table 1.

In system (5), the susceptible population is formulated by adding the human birth rate and temporarily recovered population. The susceptible human population who become infected after the transmission β_1, β_2 are reduced from the susceptible and included in the exposed population. The host population awareness rate ω_1 deals with the number of individuals in the host population who are aware of a specific disease or health issue. In the setting of infectious diseases like Zika, host population awareness is a key component that can influence disease transmission and the accomplishment of public health measures. A higher degree of awareness can lead to greater commitment to preventive measures such as vaccination, mask use, proper cleanliness, and seeking medical care on time.

As a result, the disease's propagation and its effect on individuals and communities can be reduced. The host population awareness rate can be influenced by numerous factors, such as access to accurate and reliable information, communication strategies used by health authorities, cultural beliefs, education levels, and previous encounters with similar health issues. Also, the rate of development from being exposed to a disease to becoming asymptomatic or infected can vary depending on the specific disease and individual factors. Some individuals who are exposed to a disease may develop asymptomatic infection, meaning they are infected with the infectious agent but do not exhibit any symptoms. These individuals can still transmit the disease to others, even though they may not be aware of their infection. Introducing the notation ψ_i which is a fuzzy function and defined as $i = 1$

Table 1. Parameter Description

Parameter	Description
Ξ_H	Humans birth rate
Ξ_M	Mosquitoes birth rate under natural condition
λ	Rate at which temporary recovery of the human considered as susceptible
β_1	Transmission rate between infected and susceptible humans
β_2	Transmission between infected mosquitoes and susceptible humans
κ_1	Death rate of humans under natural condition
κ_2	Mosquitoes death rate under natural condition
ω_1	Host population awareness rate
ω_2	Effective and fixed control rate of mosqouitoes
γ	Recovered rate from infected
σ_1	Rate of infected humans after exposure
σ_2	Rate of exposed mosquitoes becoming infected
μ	The probability of infected human bit by susceptible mosquitoes
Ψ	Rate of development from exposed $\mathtt{E}_{(\mathtt{h})}$ to asymptomatic $\mathcal{A}\mathtt{I}_{(\mathtt{h})}$ or infected $\mathtt{I}_{(\mathtt{h})}$
Φ	Proportion of individuals who are either asymptomatic $\mathcal{A}\mathtt{I}_{(\mathtt{h})}$ or infected $\mathtt{I}_{(\mathtt{h})}$

to 8. The ψ_i's are termed as mentioned below,

$$\begin{aligned}
\psi_1 &= \Xi_H + \lambda \mathtt{R}_{(\mathtt{h})} - \beta_1 \mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{h})} - \beta_2 \mathtt{I}_{(\mathtt{m})}\mathtt{S}_{(\mathtt{h})} - (\omega_1 + \kappa_1)\mathtt{S}_{(\mathtt{h})}, \\
\psi_2 &= \beta_1 \mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{h})} + \beta_2 \mathtt{I}_{(\mathtt{m})}\mathtt{S}_{(\mathtt{h})} - (\kappa_1 + \sigma_1)\mathtt{E}_{(\mathtt{h})}, \\
\psi_3 &= \sigma_1 \mathtt{E}_{(\mathtt{h})} - (\kappa_1 + \gamma)\mathtt{I}_{(\mathtt{h})}, \\
\psi_4 &= \Psi(1 - \Phi)\mathtt{E}_{(\mathtt{h})} - \kappa_1 \mathcal{A}\mathtt{I}_{(\mathtt{h})}, \\
\psi_5 &= \gamma \mathtt{I}_{(\mathtt{h})} - \kappa_1 \mathtt{R}_{(\mathtt{h})} + \omega_1 \mathtt{S}_{(\mathtt{h})} - \lambda \mathtt{R}_{(\mathtt{h})}, \\
\psi_6 &= \Xi_M - \mu \mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{m})} - (\omega_2 + \kappa_2)\mathtt{S}_{(\mathtt{m})}, \\
\psi_7 &= \mu \mathtt{I}_{(\mathtt{h})}\mathtt{S}_{(\mathtt{m})} - (\omega_2 + \sigma_2 + \kappa_2)\mathtt{E}_{(\mathtt{m})}, \\
\psi_8 &= \sigma_2 \mathtt{E}_{(\mathtt{m})} - (\omega_2 + \kappa_2)\mathtt{I}_{(\mathtt{m})},
\end{aligned} \tag{6}$$

For $0 < \alpha < 1$, the system of equations (5) is formulated by taking $\mathcal{W}(t)$ as $(\mathtt{S}_{(\mathtt{h})}, \mathtt{E}_{(\mathtt{h})}, \mathtt{I}_{(\mathtt{h})}, \mathcal{A}\mathtt{I}_{(\mathtt{h})}, \mathtt{R}_{(\mathtt{h})}, \mathtt{S}_{(\mathtt{m})}, \mathtt{E}_{(\mathtt{m})}, \mathtt{I}_{(\mathtt{m})})^T$ and also $\mathcal{X}(t, \mathcal{W}(t)) = (\psi_i)^T$ with the initial condition $\mathcal{W}_0 = \mathcal{W}_0 \geq 0, t \geq 0$. The system of equations (5) are expressed as,

$$^{C}D^{\alpha}\mathcal{W}(t) = \mathcal{X}(t, \mathcal{W}(t)) \tag{7}$$

4. Main Results

In this model (Fig. 1), the Zika viral spread is formulated primarily based on the factors comprising humans and mosquitoes. Also, the model involves factors such as birth rate, recovered rate, death rate, transmission rate, and awareness rate that are directly related to the transmission of the

Zika virus. Due to their generalisation in order and hereditary nature, fractional calculus is often used in important domains.

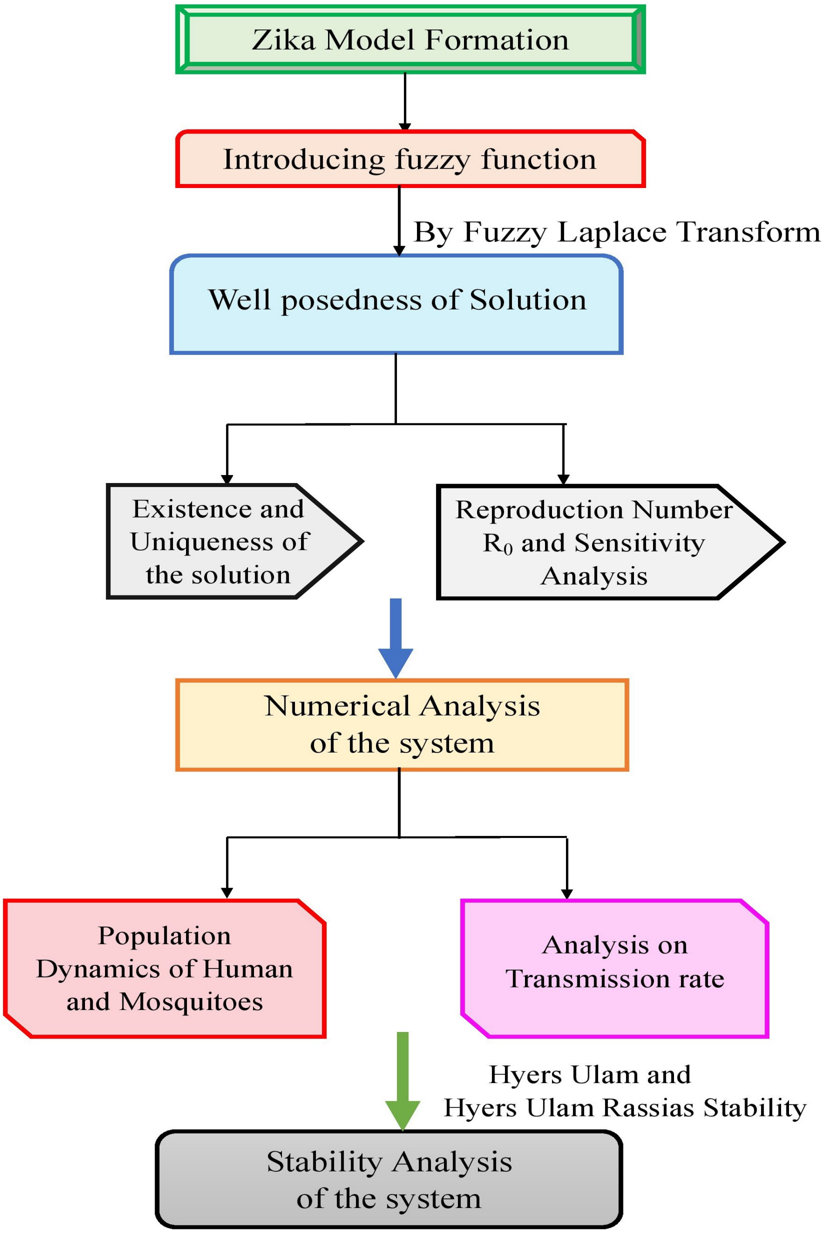

Fig. 1. Research Methodology

4.1. Existence of the Solution

For modeling the natural world, differential equations have always served as an essential tool. After creating a differential equation model of an ecological system, the researcher becomes interested in how the relevant variables evolve through time. Finding a solution and analysing its uniqueness are essential parts of modeling. Incorporating fuzzy Laplace transform in finding the solution to the proposed model under fractional order is provided as follows,

Theorem 1. *Let α be $0 < \alpha < 1$ and $t \geq 0$. There exist a solution $\mathcal{W}(t)$ for the model (6) based on* *the initial conditions* $\mathcal{W}_0 = \mathcal{W}_0$ *where* $\mathcal{W}_0 \geq 0$.

Proof. The Caputo fractional order Zika virus model is expressed as,

$$^{C}D^{\alpha}\mathcal{W}(t) = \mathcal{X}(t, \mathcal{W}(t))$$

Employing the laplace transformation on both sides, where s denotes a positive parameter

$$L[^{C}D^{\alpha}\mathcal{W}(t)] = L[\mathcal{X}(t, \mathcal{W}(t))]$$

$$s^{\alpha}\mathbf{Z}(s) \ominus \sum_{k=0}^{n-1} s^{\alpha-k-1}\mathcal{W}_0^k = L[\mathcal{X}(t, \mathcal{W}(t))],$$

$$\mathcal{W}(t) = \frac{1}{\Gamma(\alpha)} \odot \int_0^t (t-s)^{\alpha-1}\mathcal{X}(s, \mathcal{W}(s))ds \oplus \mathcal{W}_0. \tag{8}$$

An operator $\mathbf{M}$ denoted as $\mathbf{M} : \mathcal{E} \to \mathcal{E}$ by

$$(\mathbf{M}\mathcal{W})(t) = \frac{1}{\Gamma(\alpha)} \odot \int_0^t (t-s)^{\alpha-1}\mathcal{X}(s, \mathcal{W}(s))ds \oplus \mathcal{W}_0. \tag{9}$$

and the operator $\mathbf{M}$ is well-defined as the continuity of $\mathcal{X}$ is obvious. □

Assumption 1. *Consider the continuous solution* $\mathcal{W}, \bar{\mathcal{W}}$*, and let* S *be a constant such that*

$$\mathfrak{D}\big[\mathcal{X}(t, \mathcal{W}(t)), \mathcal{X}(t, \bar{\mathcal{W}}(t))\big] \leq S\mathfrak{D}[\mathcal{W}(t) - \bar{\mathcal{W}}(t)] \tag{10}$$

Lemma 1. *Consider* $\bar{\mathcal{W}} = (\bar{\mathtt{S}}_{(\mathtt{h})}, \bar{\mathtt{E}}_{(\mathtt{h})}, \bar{\mathtt{I}}_{(\mathtt{h})}, \bar{\mathcal{A}}\mathtt{I}_{(\mathtt{h})}, \bar{\mathtt{R}}_{(\mathtt{h})}, \bar{\mathtt{S}}_{(\mathtt{m})}, \bar{\mathtt{E}}_{(\mathtt{m})}, \bar{\mathtt{I}}_{(\mathtt{m})})$. *With the aid of operator* (9) *and the hypothesis* (10)*, the fuzzy fuction* $\mathcal{W}(t)$ *satisfies the Lipschitz condition provided that for all* $\mathcal{W}(t)$ *and* $\bar{\mathcal{W}}(t)$ *there exist* $\mathcal{L} > 0$ *satisfying,*

$$\mathfrak{D}[\mathbf{M}\mathcal{W}(t), \mathbf{M}\bar{\mathcal{W}}(t)] \leq \mathcal{L}\mathfrak{D}[\mathcal{W}(t) - \bar{\mathcal{W}}(t)]. \tag{11}$$

provided that $\mathcal{L} = S\dfrac{b^{\alpha+1}}{\Gamma(\alpha)}$

Proof.

$$\begin{aligned}
\mathfrak{D}[\mathbf{M}\mathcal{W}(t), \mathbf{M}\bar{\mathcal{W}}(t)] =& \mathfrak{D}\left[\frac{1}{\Gamma(\alpha)}\left(\int_0^t (t-s)^{\alpha-1} \odot \mathcal{X}(s, \mathcal{W}(s))ds\right) + \mathcal{W}_0,\right. \\
&\left.\frac{1}{\Gamma(\alpha)}\left(\int_0^t (t-s)^{\alpha-1} \odot \mathcal{X}(s, \bar{\mathcal{W}}(s))ds\right) + \bar{\mathcal{W}}_0\right] \\
=& \frac{1}{\Gamma(\alpha)}\mathfrak{D}\left[\int_0^t (t-s)^{\alpha-1} \odot \mathcal{X}(s, \mathcal{W}(s))ds, \int_0^t (t-s)^{\alpha-1} \odot \mathcal{X}(s, \bar{\mathcal{W}}(s))ds\right] \\
&\oplus \mathfrak{D}[\mathcal{W}_0, \bar{\mathcal{W}}_0] \\
\leq& \frac{b^{\alpha}}{\Gamma(\alpha)}\mathfrak{D}\left[\mathcal{X}(t, \mathcal{W}(t)), \mathcal{X}(t, \bar{\mathcal{W}}(t))\right] \\
\leq& S\frac{b^{\alpha+1}}{\Gamma(\alpha)}\mathfrak{D}\big[\mathcal{W}(t), \bar{\mathcal{W}}(t)\big] \\
\leq& \mathcal{L}\mathfrak{D}\big[\mathcal{W}(t), \bar{\mathcal{W}}(t)\big]
\end{aligned}$$

With the following conideration the initial value is considered as $\mathfrak{D}[\mathcal{W}_0, \bar{\mathcal{W}}_0] = 0$. Further, the $t \in [0, b]$ leads to $\int_0^t (t-s)^{\alpha-1}ds < b^{\alpha}$. The Lipschitz condition is closely related to the system's stability. If a model holds the Lipschitz condition, it suggests that the system's behavior remains bounded and does not exhibit excessive oscillations or divergent behavior. This characteristic is essential for comprehending a system's long-term behavior and predictability of a system. □

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Theorem 2. *There exist an unique solution for the fractional order Zika virus model (7) under Caputo sense, if*

$$S\frac{b^{\alpha+1}}{\Gamma(\alpha)} < 1 \tag{12}$$

holds.

Proof. Assuming that there exist another solution $\mathcal{W}^*(t)$ satisfying (7)

$$\begin{aligned}
\mathfrak{D}[\mathcal{W}(t), \mathcal{W}^*(t)] =& \mathfrak{D}\Bigg[\frac{1}{\Gamma(\alpha)}\left(\int_0^t (t-s)^{\alpha-1} \odot \mathcal{X}(s, \mathcal{W}(s))ds\right) + \mathcal{W}_0, \\
& \frac{1}{\Gamma(\alpha)}\left(\int_0^t (t-s)^{\alpha-1} \odot \mathcal{X}(s, \mathcal{W}^*(s))ds\right) + \mathcal{W}^*{}_0\Bigg] \\
\leq& \frac{b^\alpha}{\Gamma(\alpha)} \odot \mathfrak{D}\Big[\mathcal{X}(t, \mathcal{W}(t)), \mathcal{X}(t, \mathcal{W}^*(t))\Big] \\
\leq& S\frac{b^{\alpha+1}}{\Gamma(\alpha)}\mathfrak{D}\big[\mathcal{W}(t), \mathcal{W}^*(t)\big] \\
\leq& \mathcal{L}\mathfrak{D}\big[\mathcal{W}(t), \mathcal{W}^*(t)\big]
\end{aligned}$$

Furthermore, it implies that

$$\mathcal{L}\mathfrak{D}\big[\mathcal{W}(t), \mathcal{W}^*(t)\big] \geq 0 \tag{13}$$

From (12) and since the equation (13) is true which results in $\mathfrak{D}\big[\mathcal{W}(t), \mathcal{W}^*(t)\big] = 0$. Thus, $\mathcal{W}(t) = \mathcal{W}^*(t)$. Hence, the solution of the fractional order Zika virus is unique. □

4.2. Reproduction Number $\mathcal{R}_0$ and Sensitivity Analysis

In the mathematical framework of viral infections, a disease-free equilibrium is attained when there is no infected individual in the population. In this state of equilibrium, the virus will not be actively spreading out, and there will be no more infected people. It denotes a steady state in which the disease has been successfully controlled or completely eradicated from the population. Disease-free equilibrium is a crucial concept in understanding the epidemiology and mathematical modeling of infectious diseases as it aids in the control strategies of diseases. The Zika virus-free equilibrium for this model is given as $\mathcal{D}_0 = (\mathtt{S}_{(\mathtt{h})}{}^0, \mathtt{E}^0_{(\mathtt{h})}, \mathtt{I}^0_{(\mathtt{h})}, \mathcal{A}\mathtt{I}^0_{(\mathtt{h})}, \mathtt{S}^0_{(\mathtt{m})}, \mathtt{E}^0_{(\mathtt{m})}) = (1, 0, 0, 0, 0, 0)$.

In studies related to epidemiology, the basic reproduction number is a vital concept that provides the number of secondary infections caused by a single infected person in an infection-free environment. It is extensively utilized to evaluate an outbreak's severity and the possibility of disease spread. With the aid of the Next-generation matrix [22], the basic reproduction number $\mathcal{R}_0$ for the model (5) is computed. Consider $y = (\mathtt{E}_{(\mathtt{h})}, \mathtt{I}_{(\mathtt{h})}, \mathcal{A}\mathtt{I}_{(\mathtt{h})}, \mathtt{E}_{(\mathtt{m})}, \mathtt{I}_{(\mathtt{m})})$. With $\mathcal{F}$ and $\mathcal{V}$ being the infection and recovery rates respectively, which is formulated using $\mathcal{F} = \dfrac{\partial F_i}{\partial y_j}$ and $\mathcal{V} = \dfrac{\partial V_i}{\partial y_j}$.

$$\mathcal{F} = \begin{bmatrix} 0 & \beta_1 & 0 & 0 & \beta_2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} \kappa_1 + \sigma_1 & 0 & 0 & 0 & 0 \\ -\sigma_1 & \gamma + \kappa_1 & 0 & 0 & 0 \\ -\Psi(1-\Phi) & 0 & \kappa_1 & 0 & 0 \\ 0 & 0 & 0 & (\kappa_2 + \sigma_2 + \omega_2) & 0 \\ 0 & 0 & 0 & -\sigma_2 & \kappa_2 + \omega_2 \end{bmatrix}$$

Thus, the matrix obtained by next generation method is,

$\mathcal{F}\mathcal{V}^{-1} =$

$$\begin{bmatrix} \frac{\beta_1\sigma_1}{(\gamma+\kappa_1)(\kappa_1+\sigma_1)} & \frac{\beta_1}{\gamma+\kappa_1} & 0 & \frac{\beta_2\sigma_2}{(\kappa_2+\omega_2)(\kappa_2+\sigma_2+\omega_2)} & \frac{\beta_2}{\kappa_2+\omega_2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\mu\sigma_1}{(\gamma+\kappa_1)(\kappa_1+\sigma_1)} & \frac{\mu}{\gamma+\kappa_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The basic reproduction for the system is obtained through,

$$\mathcal{R}_0 = \rho(\mathcal{F}\mathcal{V}^{-1}) = \frac{\beta_1\sigma_1(\kappa_2+\omega_2)(\kappa_2+\omega_2+\sigma_2) + \sqrt{\sigma_1(\kappa_2+\omega_2)}\sqrt{\kappa_2+\omega_2+\sigma_2}\sqrt{4\mu\beta_2(\gamma+\kappa_1)(\kappa_1+\sigma_1)\sigma_2+\beta_1^2\sigma_1(\kappa_2+\omega_2)(\kappa_2+\omega_2+\sigma_2)}}{2(\gamma+\kappa_1)(\kappa_1+\sigma_1)(\kappa_2+\omega_2)(\kappa_2+\omega_2+\sigma_2)} \tag{14}$$

Validation increases the confidence in the model's capacity to deliver correct estimations or insights. It is an important phase in the scientific process that encourages reliable and effective research results. To validate a model and determine its robustness, the sensitivity analysis for the model is essential. The benefit of performing sensitivity analysis helps in understanding the impact of parameters on the model [23]. The sensitivity index approach is used for determining the measure of sensitive parameters that influence the reproduction number $\mathcal{R}_0$. The sensitivity index for the parameters of $\mathcal{R}_0$ are computed using

$$\mathbb{S} = \frac{Q_i}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial Q_i}$$

where Q_i denotes the parameters. The parameters that influence the reproduction number are $\sigma_1, \sigma_2, \kappa_1, \kappa_2, \beta_1, \beta_2, \omega_2, \gamma$ and μ, and its sensitivity index are computed as follows,

$$\begin{aligned}
\mathbb{S}_{\sigma_1} &= \frac{\sigma_1}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \sigma_1} = 0.000501876 > 0, & \mathbb{S}_{\sigma_2} &= \frac{\sigma_2}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \sigma_2} = 0.0322437 > 0, \\
\mathbb{S}_{\kappa_1} &= \frac{\kappa_1}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \kappa_1} = -1.46575 \times 10^6 < 0, & \mathbb{S}_{\kappa_2} &= \frac{\kappa_2}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \kappa_2} = -4.4862 \times 10^7 < 0, \\
\mathbb{S}_{\beta_1} &= \frac{\beta_1}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \beta_1} = 4.22378 \times 10^8 > 0, & \mathbb{S}_{\beta_2} &= \frac{\beta_2}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \beta_2} = 3.08165 \times 10^7 > 0, \\
\mathbb{S}_{\omega_2} &= \frac{\omega_2}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \omega_2} = -0.00322469 < 0, & \mathbb{S}_{\gamma} &= \frac{\gamma}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \gamma} = -4.51972 \times 10^8 < 0, \\
\mathbb{S}_{\mu} &= \frac{\mu}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial \mu} = 3.08165 \times 10^7 > 0.
\end{aligned}$$

where $\mathbb{S}_{\sigma_1}, \mathbb{S}_{\sigma_2}, \mathbb{S}_{\kappa_1}, \mathbb{S}_{\kappa_2}, \mathbb{S}_{\beta_1}, \mathbb{S}_{\beta_2}, \mathbb{S}_{\omega_2}, \mathbb{S}_{\gamma}$ and $\mathbb{S}_{\mu}$ denotes the sensitivity index of the parameters $\sigma_1, \sigma_2, \kappa_1, \kappa_2, \beta_1, \beta_2, \omega_2, \gamma$ and μ. From the above analysis, the reproduction number can be increased due to the increase of parameters such as $\sigma_1, \sigma_2, \beta_1, \beta_2$ and μ. Subsequently, the decrease in $\mathcal{R}_0$ can be achieved by decrease in $\kappa_1, \kappa_2, \omega_2$ and γ

5. Numerical Simulation

Mathematical simulation is a significant technique for establishing effective control methods and forecasting disease outbreaks. Mathematical models are helpful tools for evaluating epidemic situations and offer a direction for enacting the proper countermeasures [24]. This part is dedicated to evaluating the proposed model numerically. The parameter values provided in Table 2 are taken into consideration for the numerical solution of system (5) and are referred from [25], [26].

Table 2. Numerical values of Parameters

Parameter	Numerical value/week	Parameter	Numerical value/week
Ξ_H	0.014957	Ξ_M	119.96
λ	0.378	β_1	0.01599
β_2	0.00014874	κ_1	0.00019204
κ_2	0.020531	ω_1	0.35809
ω_2	0.00071429	γ	0.07098
σ_1	0.35809	σ_2	0.020706
μ	0.08134	Ψ	1.4
Φ	0.091		

For the numerical computation of the model, we employ the following initial conditions, $\mathtt{S}_{(\mathtt{h})_0} = 5000, \mathtt{E}_{(\mathtt{h})_0} = 2000, \mathtt{I}_{(\mathtt{h})_0} = 1000, \mathcal{A}\mathtt{I}_{(\mathtt{h})_0} = 500, \mathtt{R}_{(\mathtt{h})_0} = 1000, \mathtt{S}_{(\mathtt{m})_0} = 300, \mathtt{E}_{(\mathtt{m})_0} = 100,$ and $\mathtt{I}_{(\mathtt{m})_0} = 100$.

5.1. Human Population Dynamics

The study related to virus and their effects on human society using mathematical models has advanced in the public health sector and is used in foreseeing vector population dynamics [27]. Different fractional order α provides different fractional differentiation of the system. The graphical representation of dynamics of the susceptible $\mathtt{S}_{(\mathtt{h})}(t)$ and exposed $\mathtt{E}_{(\mathtt{h})}(t)$ human population are provided in (Fig. 2) and (Fig. 3) for the order $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9$, respectively. By analysing the graphs, it is observed that the susceptible dynamics can be raised by boosting the recovery rate and further it is decreased by the change in the range of human and mosquito transmission. By altering viral transmission, the exposed human population exhibits increasing dynamics. It is observed that the transmission by humans increases the dynamic range than the transmission by mosquitoes. The population exposed to the virus can be reduced to a greater extent by preventing the spread of the virus from human to human.

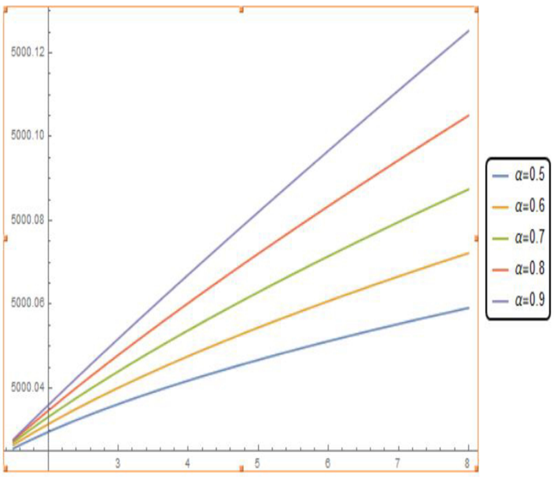

Fig. 2. Population dynamics of $\mathtt{S}_{(\mathtt{h})}(t)$

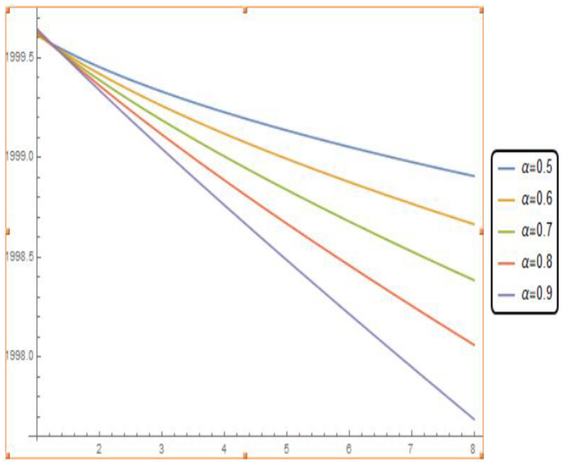

Fig. 3. Population dynamics of $\mathtt{E}_{(\mathtt{h})}(t)$

Based on the spread of infection, the dynamics of infected $\mathtt{I}_{(\mathtt{h})}(t)$, asymptomatically infected $\mathcal{A}\mathtt{I}_{(\mathtt{h})}(t)$ and recovered $\mathtt{R}_{(\mathtt{h})}(t)$ human population are provided in (Fig. 4), (Fig. 5) and (Fig. 6), respectively, for $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9$. It is observed from the above graphs, that the dynamics of the infected and asymptomatically infected population is directly related to the rate of exposure. Controlling the exposed population has a greater impact on reducing the infected population.

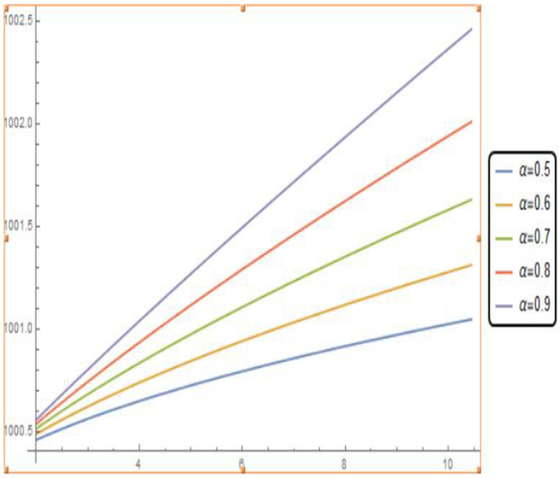

Fig. 4. Population dynamics of $\mathtt{I}_{(\mathtt{h})}(t)$

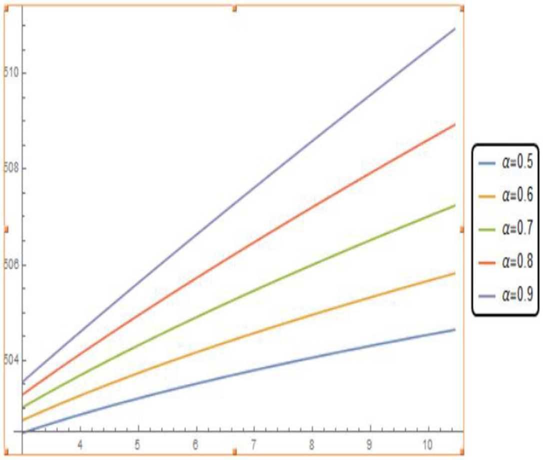

Fig. 5. Population dynamics of $\mathcal{A}\mathtt{I}_{(\mathtt{h})}(t)$

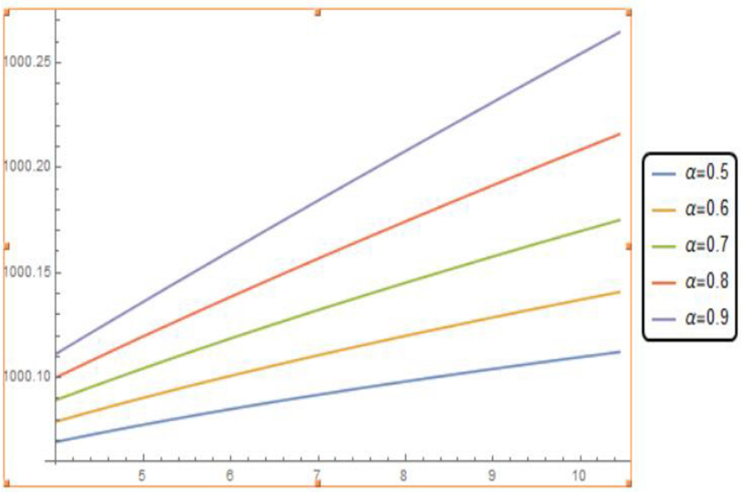

Fig. 6. Population dynamics of $\mathtt{R}_{(\mathtt{h})}(t)$

5.2. Dynamics of Mosquito Population

The dynamics of mosquitoes are extremely important in understanding the spreading of Zika virus. Understanding and managing mosquito populations is essential in preventing viral transmission and reducing the risk to human populations. The solution path of mosquitoes is analysed in this section. The solution path of susceptible $\mathtt{S}_{(\mathtt{m})}(t)$, exposed $\mathtt{E}_{(\mathtt{m})}(t)$ and infected $\mathtt{I}_{(\mathtt{m})}(t)$ mosquitoes are provided graphically in (Fig. 7), (Fig. 8) and (Fig. 9) respectively. By examining the graph (Fig. 8), it is noticed that the exposed mosquito path can be reduced by increasing the control rate, the death rate of mosquitoes and the rate of viral spread among the mosquitoes exposed to infection. Also, by analysing the infected mosquito dynamics (Fig. 9), it implies that an increase in exposed mosquitoes causes a rise in the dynamics of infected mosquitoes. The reduced amount of exposed and infection-affected mosquitoes triggers an overall decrease in viral transmission by mosquitoes.

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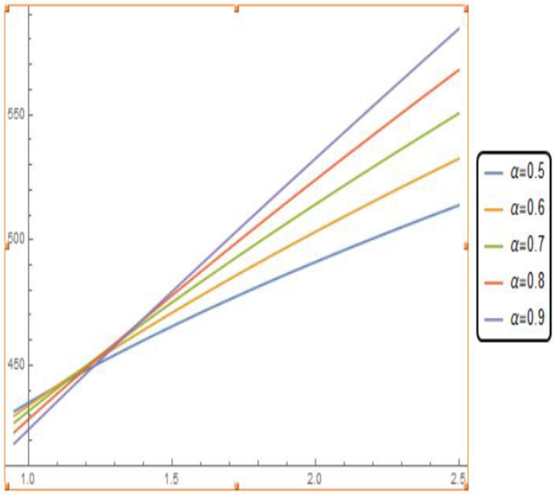


Fig. 7. Susceptible mosquitoes $\mathtt{S}_{(\mathtt{m})}(t)$

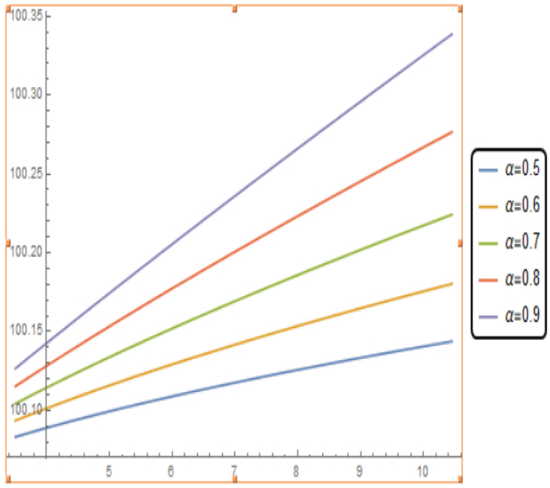


Fig. 8. Exposed mosquitoes $\mathtt{E}_{(\mathtt{m})}(t)$

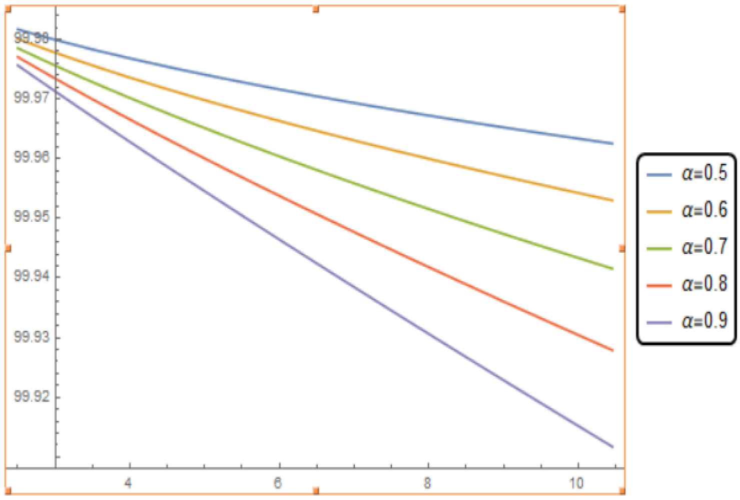


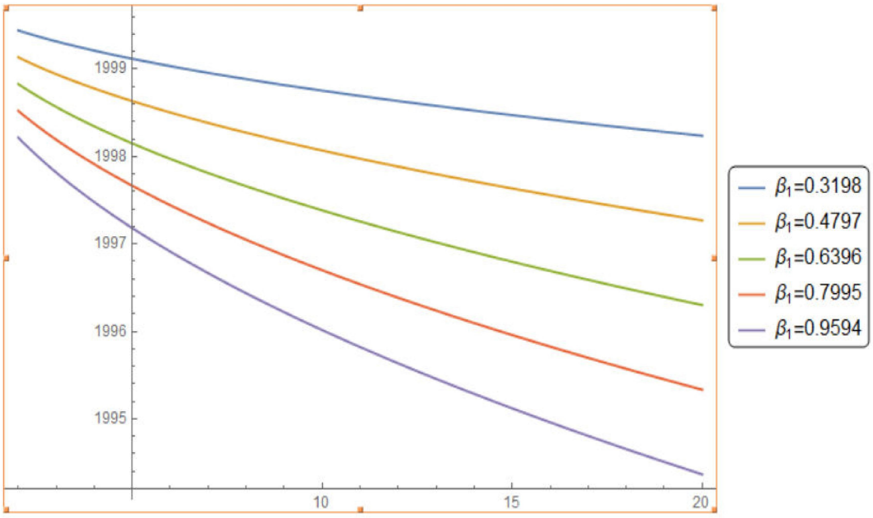
Fig. 9. Infected mosquitoes $\mathtt{I}_{(\mathtt{m})}(t)$

6. Result and Discussion

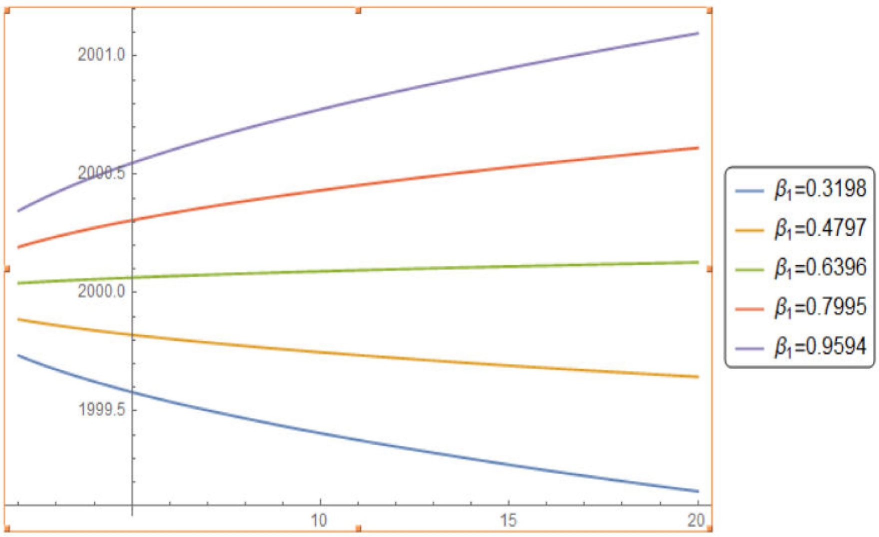
In order to study the transmission dynamics of the proposed Zika virus model, the transmission parameters are examined. The Zika virus can transmit from person to person in several ways, including mother to foetus during pregnancy, blood transfusion, and sexual contact. In the first part, the transmission between human - humans is studied for different fractional orders in the form of numerical data as well as graphical representation. For a detailed analysis of the effect caused by human to human transmission, the transmission parameter β_1 is utilized and varied fractionally to observe the changes occurring in the solution path of the susceptible and exposed human population. The graphical result (Fig. 10) and its numerical observation in Table 3 of the aforementioned transmission are presented for understanding. It is observed that the susceptible values gradually fall as the transmission rate increases fractionally, indicating a decrease in the susceptible individuals. Further, it is also observable that there is a slight increase in the exposed values, which shows a gradual rise in the amount of people exposed to the transmission.

Table 3. Approximate solution of Susceptible and Exposed population

β_1 value	Susceptible	Exposed
0.3198	1999.2	1999.6
0.4797	1998.6	1999.8
0.6396	1998.2	2000.1
0.7995	1997.6	2000.4
0.9594	1997.2	2000.6



(a) Solution path of Susceptible population $S_{(h)}$



(b) Solution path of Exposed population $E_{(h)}$

Fig. 10. Human-to-human viral transmission in the Susceptible and Exposed population

Secondly, the transmission between mosquitoes - humans are studied for different fractional order in the form of numerical data as well as a graphical representation. For the purpose of gaining a better understanding of the impact of mosquito-to-human transmission, the transmission parameter β_2 is reduced fractionally to examine variations in the solution path of susceptible and exposed human populations. The variations are represented visually as graphical representation in (Fig. 11) and numerically in Table 4. It is observable that even a minute fractional reduction in transmission rate produces a rise in susceptible and a decrease in the exposed population (Fig. 12).

Table 4. Approximate solution of Susceptible and Exposed population

β_2 value	Susceptible	Exposed
0.0133866	2000.05	1998.5
0.007437	2000.07	1997
0.0044622	2000.075	1996.5
0.0014874	2000.085	1996.5
0.0029748	2000.08	1996

Based on the above comparisons, the human to human transmission, that is, β_1, increased at a fractional level results in fractional level rise in the exposed population growth and a fractional decrease in the susceptible population. It is also noted that by varying β_2, that is, mosquitoes to human transmission, a slight fractional increase in transmission rate causes an integer level increase in the exposed population. Two insights emerge from the comparison as well as from the model. Firstly, in terms of transmission rate, the human to human transmission is higher than mosquitoes to humans. Secondly, even though the transmission between mosquitoes and humans is less common, their impact on exposed humans is higher than human-human transmission. Thus based on this model and transmission rates, preventing mosquito transmission can contribute to the reduction in the spread of the Zika virus at a fractional level.

In [25], the graphical simulation of the fractional Zika model indicates that the lower contact rate and fractional order, are essential for substantially reducing infection. The authors of [10] proposed that the integer and fractional Zika model behaves differently on incorporating numerical values. The numerical outcomes for the system are obtained using the Euler technique. The research contribution of this work provides a clear visual and numerical representation of the effect of transmission rate. The transmission occurring between human to humans and the transmission between mosquitoes to humans are explained in detail. Further, the accuracy of the Euler method is highly dependent on the chosen step size. A smaller step size generally improves accuracy but increases computational time. Finding an appropriate step size that balances accuracy and computational efficiency can be challenging. Whereas, the Laplace transform has fewer computations and helps assess complex systems with time-varying coefficients, making it beneficial for analyzing dynamic systems. Viral infections and viruses, such as the Zika virus, have the potential to cause birth defects or neurological impairments in individuals and populations. Through minimization of transmission, we can lower the possibility of the Zika virus spread and reduce the secondary infections caused by it. Community initiatives are essential [28] in eradicating mosquito breeding places, to support local government and public health activities.

7. Stability

The Hyers-Ulam stability is an effective tool for demonstrating stability in various mathematical domains, including functional analysis, differential equations, and dynamical systems. It has been widely employed in many fields of mathematics, notably in the study of differential equations stability, and functional equations stability.

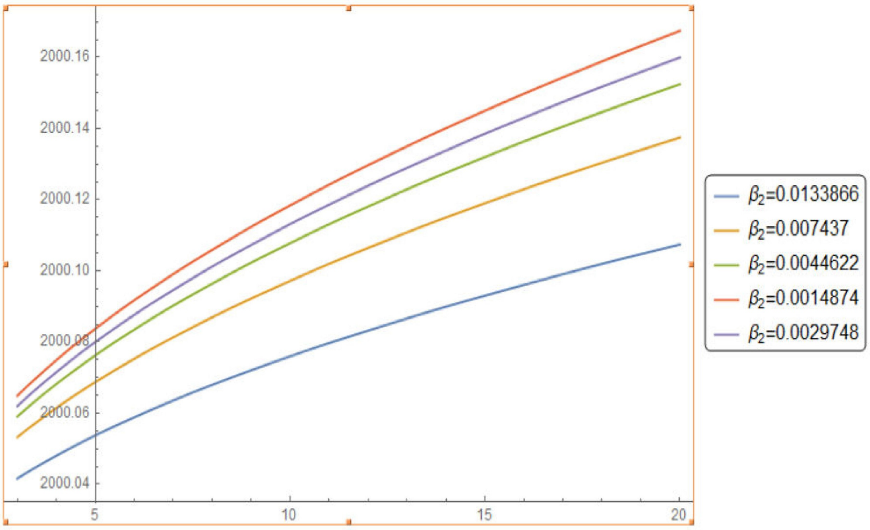
Assumption 2. *(A1) Assume that $\epsilon > 0$, $|^{C}D^{\alpha}\mathcal{W}(t) - \mathcal{X}(t, \mathcal{W}(t))| \leq \epsilon$. and $t \in [0, b]$ [34].*

Definition 7. *Let $\mathcal{C}_{\mathcal{E}} > 0$ be the constant and $\epsilon > 0$. As for every solution $\mathcal{W}^*(t)$ in $\mathcal{E}$, that holds*

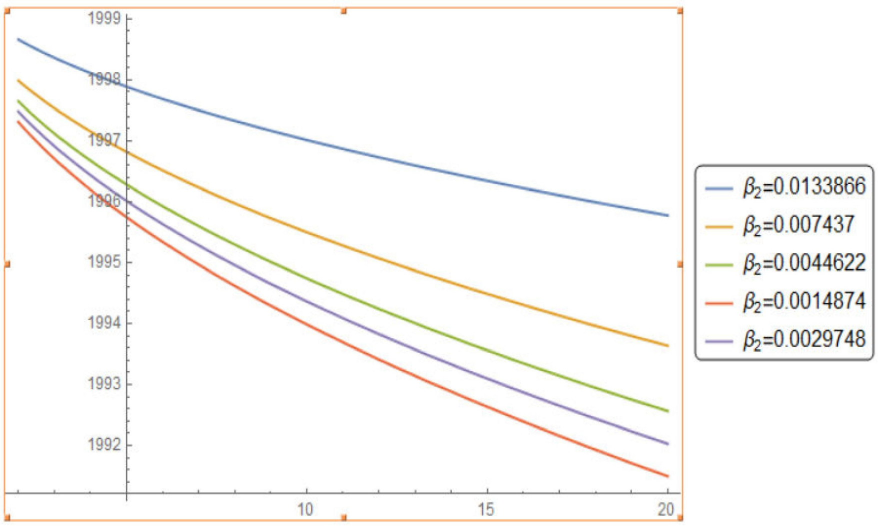
(A1) there exists a solution $\mathcal{W}(t)$ in $\mathcal{E}$ such that

$$\mathfrak{D}\big[\mathcal{W}^*(t), \mathcal{W}(t)\big] \leq \mathcal{C}_{\mathcal{E}}\epsilon \tag{15}$$

Then the fractional order system of Zika virus model (5) is Hyers-Ulam stable.



(a) Dynamics of Susceptible population $\mathtt{S}_{(\mathtt{h})}$



(b) Dynamics of Exposed population $\mathtt{E}_{(\mathtt{h})}$

Fig. 11. Mosquitoes-to-human viral transmission in the Susceptible and Exposed population

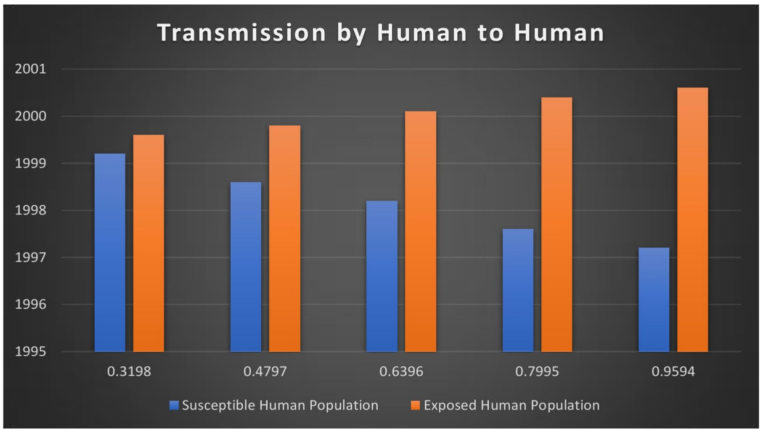
7.1. Ulam Hyers Stability

Theorem 3. *Assume that* $\mathrm{D} = \dfrac{b^{\alpha}}{\Gamma(\alpha+1)}$ *[29]. Consider (15) and the result of Lemma 1 holds and also* $1 - \mathrm{D}\mathcal{L} > 0$. *Then the proposed Zika viral model (5) satisfies Ulam HyersStablility under fractional order.*

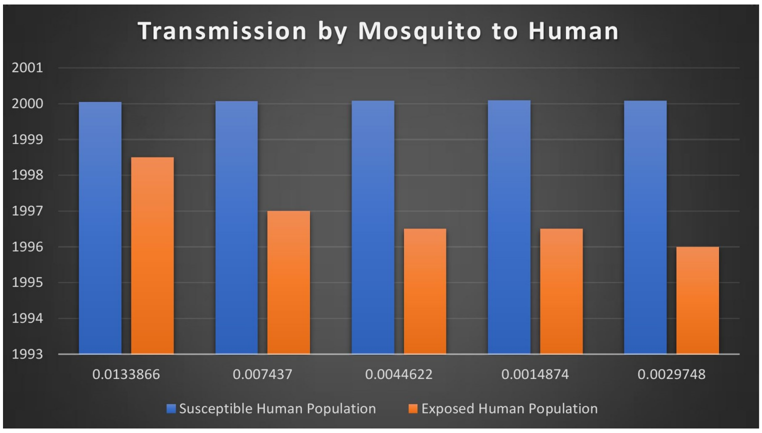
Proof. Suppose the solution of (7) be denoted as $\mathcal{W}(t)$ and let the solution for (15) be $\mathcal{W}^*(t)$. For $\epsilon > 0$, and using the Laplace transform method, the subsequent steps are obtained,

$$\begin{aligned}\mathfrak{D}[\mathcal{W}^*, \mathcal{W}]_{\mathcal{E}} &= \mathfrak{D}\big[\mathcal{W}^*(t), \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathcal{X}(s,\mathcal{W}(s))ds \oplus \mathcal{W}_0\big]\\ &= \mathfrak{D}\big[\mathcal{W}^*(t), \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathcal{X}(s,\mathcal{W}(s))ds \oplus \mathcal{W}_0 \oplus\\ &\int_0^t (t-s)^{\alpha-1}\frac{1}{\Gamma(\alpha)}\mathcal{X}(s,\bar{\mathcal{W}}(s))ds \ominus \int_0^t (t-s)^{\alpha-1}\frac{1}{\Gamma(\alpha)}\mathcal{X}(s,\bar{\mathcal{W}}(s))ds\big]\\ &\leq \mathrm{D}\epsilon + \mathrm{D}\mathcal{L}\mathfrak{D}\big[\mathcal{W}^*, \mathcal{W}\big]_{\mathcal{E}}\\ &\leq \frac{\mathrm{D}}{1-\mathrm{D}\mathcal{L}}\epsilon\\ \mathfrak{D}[\mathcal{W}^*, \mathcal{W}]_{\mathcal{E}} &\leq \mathrm{K}\epsilon.\end{aligned}$$

where, $\mathrm{K} \leq \dfrac{\mathrm{D}}{1-\mathrm{D}\mathcal{L}}$. □



(a) Human to human transmission



(b) Mosquitoes to human transmission

Fig. 12. Graphical representation of variations occured in Susceptible and Exposed human population due to human and mosquito transmission

7.2. Ulam Hyers Rassias Stability

The Ulam Hyers Rassias stability has been extended in many ways which led to numerous research papers and advances in mathematics.

Definition 8. *Consider the equation $\mathcal{W}(t)$, attaining*

$$|^{C}D^{\alpha}\mathcal{W}(t) - \mathcal{X}(t, \mathcal{W}(t))| \leq \epsilon\upsilon(t), for \quad t \in [0, b] \tag{16}$$

is denoted as Ulam Hyers Rassias stable, if a constant $\mathcal{C}_{\mathcal{E}} > 0$ exists and $\upsilon : [0, b] \to [0, \infty)$ a function exists for any possible solution $\mathcal{W}^*(t)$ of the equation (5) satisfying

$$\mathfrak{D}[\mathcal{W}^*(t) - \mathcal{W}(t)] \leq \mathcal{C}_{\mathcal{E}}\epsilon\upsilon(t) \tag{17}$$

Theorem 4. *Suppose $\mathcal{X}$ and υ be continuous functions and are termed as $\mathcal{X} : [0, b] \times \mathbb{R}^7 \to \mathbb{R}^7$ and $\upsilon : \mathbb{R} \to \mathbb{R}$. Let $\mathcal{J} \leq 1 - \mathrm{D}\mathcal{L}_{\mathcal{E}}$ [35]. Then the system of fractional equations related to Zika virus model (5) is Hyers-Ulam-Rassias Stable.*

Proof. Considering $\mathcal{W}^*(t)$ as the solution for the (16) inequality. Further,

$$-\epsilon\upsilon(t) \leq {}^{C}D^{\alpha}\mathcal{W}(t) - \mathcal{X}(t, \mathcal{W}(t)) \leq \epsilon\upsilon(t)$$

Using laplace transform provides,

$$\begin{aligned} L[-\epsilon\upsilon(t)] \leq L[^{C}D^{\alpha}\mathcal{W}(t) - \mathcal{X}(t, \mathcal{W}(t))] \leq L[\epsilon\upsilon(t)], \\ -\epsilon\Upsilon(s) \leq \mathcal{W}(t) - \mathcal{W}_0 - \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathcal{X}(s, \mathcal{W}(s))ds \leq \epsilon\Upsilon(s), \\ |\mathcal{W}(t) - \mathcal{W}_0 - \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathcal{X}(s, \mathcal{W}(s))ds| \leq \epsilon\Upsilon(s). \end{aligned}$$

Consider,

$$\begin{aligned} \mathfrak{D}[\mathcal{W}^*, \mathcal{W}]_{\mathcal{E}} &= \sup_{t\in[0,b]} |\mathcal{W}^*(t) - \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathcal{X}(s, \mathcal{W}(s))ds - \mathcal{W}_0| \\ &= \sup_{t\in[0,b]} |\mathcal{W}^*(t) - \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathcal{X}(s, \mathcal{W}(s))ds - \mathcal{W}_0 \pm \\ &\int_0^t (t-s)^{\alpha-1}\frac{1}{\Gamma(\alpha)}\mathcal{X}(s, \mathcal{W}^*(s))ds| \\ &\leq \epsilon\Upsilon(s) + \mathrm{D}L_{\mathcal{E}}\mathfrak{D}[\mathcal{W}^*, \mathcal{W}]_{\mathcal{E}} \leq \mathcal{J}\epsilon\Upsilon(s) \end{aligned}$$

□

8. Conclusion

Population dynamics provides a quantitative framework for understanding how populations of organisms change over time. By incorporating population dynamics into biological modeling, we gain insights into species interactions, predict population trends, assess population viability, identify drivers of population change, and optimize management strategies. The mathematical model under fuzzy fractional order for the Zika virus is developed in this study. The solution for the model is derived and verified for its uniqueness using the fuzzy Laplace transform. Also, the basic reproduction number and sensitivity analysis are performed on the model for validation. Considering various values of fractional order helps in determining disease prevalence. The disease dynamics are derived from values computed using the laplace transform and graphically depicted. Further, the viral transmission between humans and transmission from mosquitoes to humans are examined and compared both numerically and graphically. From the comparison, it is evident that under the proposed model the transmission through mosquitoes has a higher impact in fractional level than the transmission through humans. Ulam Hyers and Ulam Hyers Rassias stability are used in evaluating the stability of the proposed model. Based on the research findings, it is indicated that mosquito species are primary vectors for Zika virus transmission, policy implications should involve implementing or strengthening mosquito control measures. By including increased funding for mosquito surveillance and control programs, such as larviciding, insecticide spraying, and removal of breeding sites, the transmission of virus can be prevented. Indeed, the proposed model is not a standard model, it involves many uncertain factors such as climate, social factors and demographic factors. Mathematical modeling of Zika virus leads to new knowledge by pointing out the uncertain categories and gaps. By modeling the biological system, the constraints and areas where present knowledge is insufficient can be explored. Identification of such knowledge gaps directs further research and investigation, resulting in the acquisition of new knowledge in those areas. These varying factors limit the system from obtaining accurate results. This study can further proceed in the following direction,

1. To extend the study to large demographics and include various parameters, which helps in reducing the limitation.
2. Including the vaccination effectiveness and the dose concentration in the modeling to find the optimal solution.

The transmission dynamics and the spread of the Zika virus can be predicted by mathematical modeling. The practical implication of this study involves conducting targeted surveillance such as detecting high-risk locations, carrying out vector control measures and planning resource allocation to control epidemics. The outcome of this study assists in raising public awareness and facilitating decision-making among individuals and communities. The insights and expertise obtained from studying Zika virus transmission dynamics can be used to improve future strategies for monitoring, response, and preventative measures for other similar infections.

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